Spatial Game Theory and the Prisoners' Dilemma

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Introduction

Simulation has an important role to play in the study of emergence in complex systems. A problem of continued interest is the way that both cooperation and spatial order emerge from many component systems [1]. Spontaneous emergence[2] of spatial structure occurs in a variety of complex systems including artificial life models[3, 4]. Spatial emergence can be modeled using a game theoretic formulation[5] such as the iterated spatial prisoner dilemma[6, 7]. This scenario is based on the well-known two-player game theory model[8].

Two prisoners accused of being partners in a crime are interrogated separately and are separately given the options to stay silent (cooperate with one another) or to testify against their partner (defect). If both cooperate they both go free and each receive the payoff reward R. If the other player cooperates then defecting carries the temptation to defect payoff of T. The sucker payoff for cooperating when the other player defects, is S. If both defect they both receive the punishment payoff P.

The usual definition for the prisoner's dilemma is for:

$$T > R > P > S \quad (1)$$

This can be written as:

	Cooperator	Defector
Cooperator	$R_1 + R_2$	$S_1 + T_2$
Defector	$T_1 + S_2$	$P_1 + P_2$

(2)

where player 1 options are the rows and player 2 options are the columns. This can be rescaled so that $T = b$, $R = 1$ and $P = S = 0$. In the case of a two-player game[9], the payoff matrix for a playoff between (column) player j and (row) player i is:

	Cooperator	Defector
Cooperator	1	b
Defector	0	0

(3)

so the temptation to defect for player i when playing off against j is b.

The counter intuitive aspect of this scenario is that as a **system** of two players they receive the maximum payoff or $2 \times R$ when they both cooperate, but **individually** they can still be tempted to defect as $b = T > R$. This model then is a very simplified vehicle for studying scenarios concerning questions of good of the individual versus that of the many or of the system as a whole.

One variation of the two-payer game is for a mesh of players to to play against all of their neighbours and to subsequently adopt the strategy (**C**ooperate or **D**efect) of the neighbouring player with the highest payoff. This is known as the spatial prisoners' dilemma and it has been studied on square lattices with periodic boundary conditions. Figure 1 shows a prisoner cell surrounded by its four nearest neighbours and its additional four next-nearest neighbours.

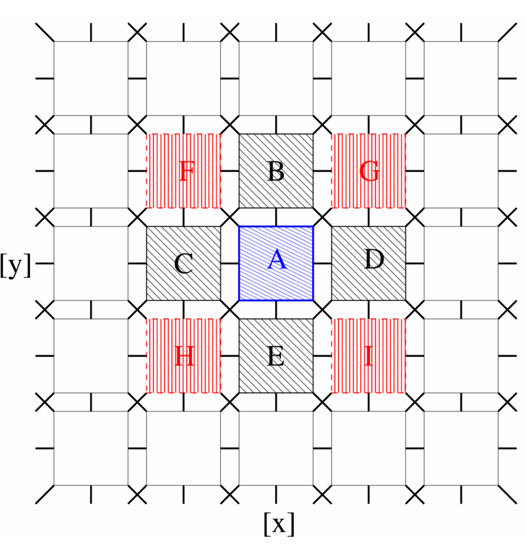

Figure 1: A player A plays-off against its 4 nearest (B-E) and 4 next-nearest neighbours (F-I).

Conceptually, each player can adopt one of two strategies - always Cooperate (C) or always defect (D) and we can study the spatial patterns of cooperators and defects and transitions between them. We adopt the colouring convention that red is a continuing defector; blue is a continuing defector; green a defector that was a cooperator; yellow a cooperator that was a defector.

We can start a model system in various configurations of cooperators and defectors and observe the dynamics and convergence (if any) of the changing patterns. A system that is all cooperators will stay stable, as will a system that is all defectors. The emergence of complex spatial structure only occurs when there is a interplay mix of the two.

We explore the properties of a system that is initialised randomly to a mix of cooperators and defectors with a fraction p of defectors. We study the final fraction f of defectors after a number of full iterations of the system. In carrying out a fine grained search in the parameter b we have found phase transitions or step functions in the long term stable values of f.

Generally f rises monotonically with b and rises as a sharp edge transition at certain b values commensurate with integer ratios. These edges were known to Nowak and May in their original work, and were ascribed to the discrete nature of the neighbour-hood model. Other authors have since studied variations of the spatial prisoner dilemma game[10] including a stochastic version and one with the addition of an artificial noise term[11, 12]. Those models seem to smear out the phase transitions making them more amenable to theoretical approaches such as mean field and percolation theory analyses.

Results

We developed two separate simulation codes to study the spatial prisoners' dilemma. One is written in **Java** and is interactive, supporting adjustment of parameters b (temptation payoff) and p (initialising fraction of defectors). It also supports editing the spatial cells directly to experiment with particular spatial arrangements of cooperator and defector player cells. This was used to generate the screen-dumps, and establish the correctness of our code as well as helping formulate explanations for particular edge transitions. A second code written in the relatively new **D** programming language is compiled and has just the cut-down core simulation algorithm and some measurement code which logs measurements to file. A **Python** scripting system[13] was used to manage the large number of independently seeded random samples and scan measurements in b parameter space.

We carried out a detailed scan in defection payoff b for various sized simulation systems and for various number of iterations. The main aim is to determine how the long term fraction of defectors f depends upon b and to identify the properties of the various phases and the edge transitions between them.

The snapshots show a periodic spatial prisoners' dilemma simulationafter 128 iteration steps from a random $50/50$ cooperator (blue)/defector(red) mix. Green squares are defectors that were cooperators, and yellow are cooperators that were defectors. At payoff parameter $b = 1.5$ (figure 2) defectors freeze out in "thin veins"; at $b = 1.6$ (figure 3) a mix of defector and cooperator clusters survive; and at $b = -1.70$ (figure 4) blockaded "islands of cooperation" can survive with no further changes.

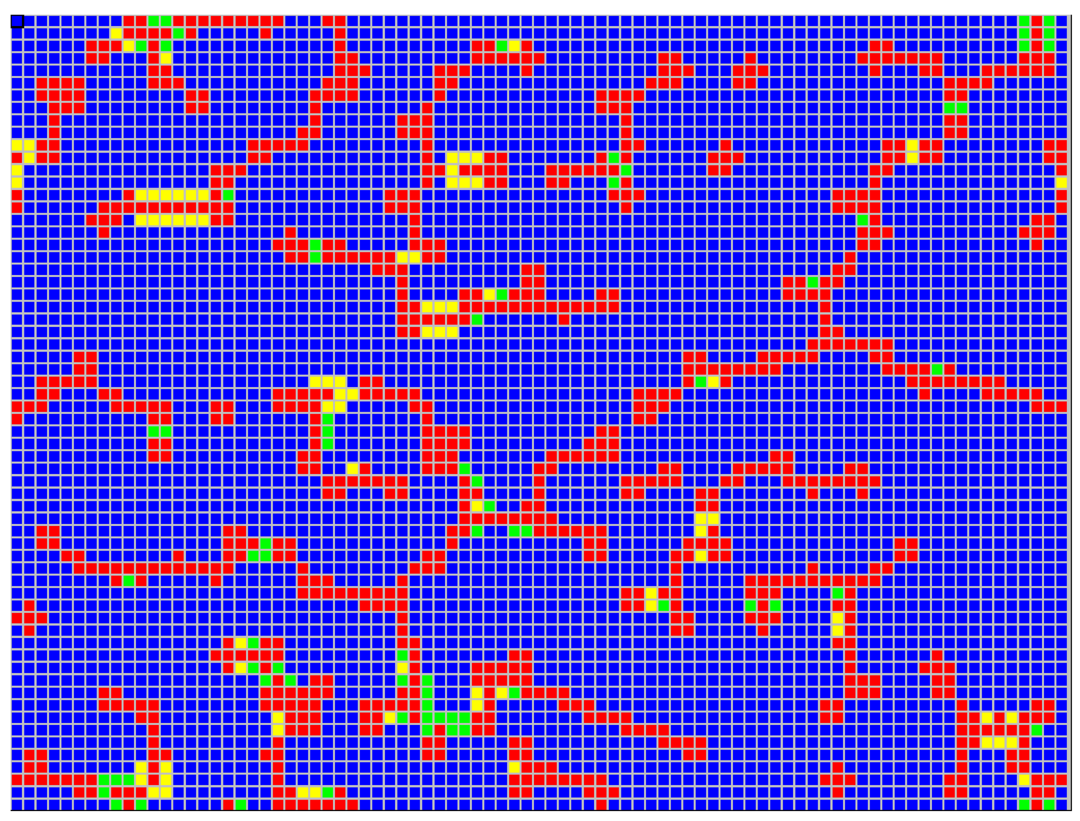

Figure 2: Payoff b=1.50

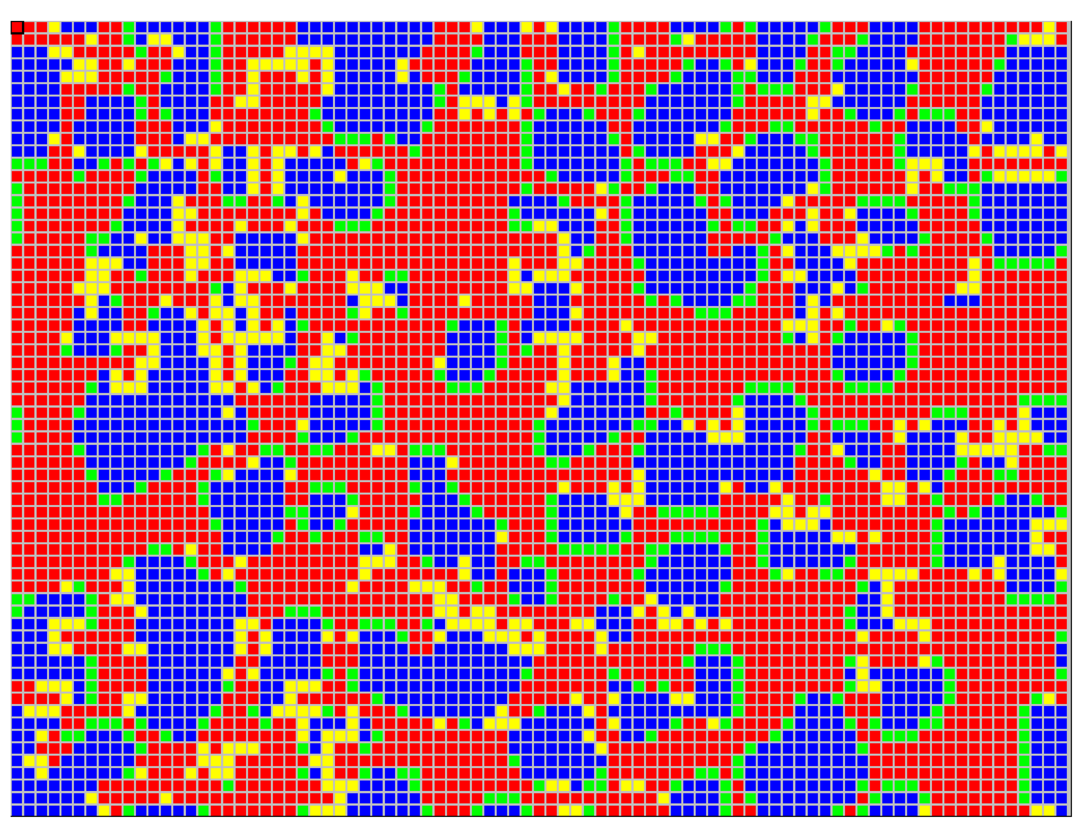

Figure 3: Payoff b=1.60

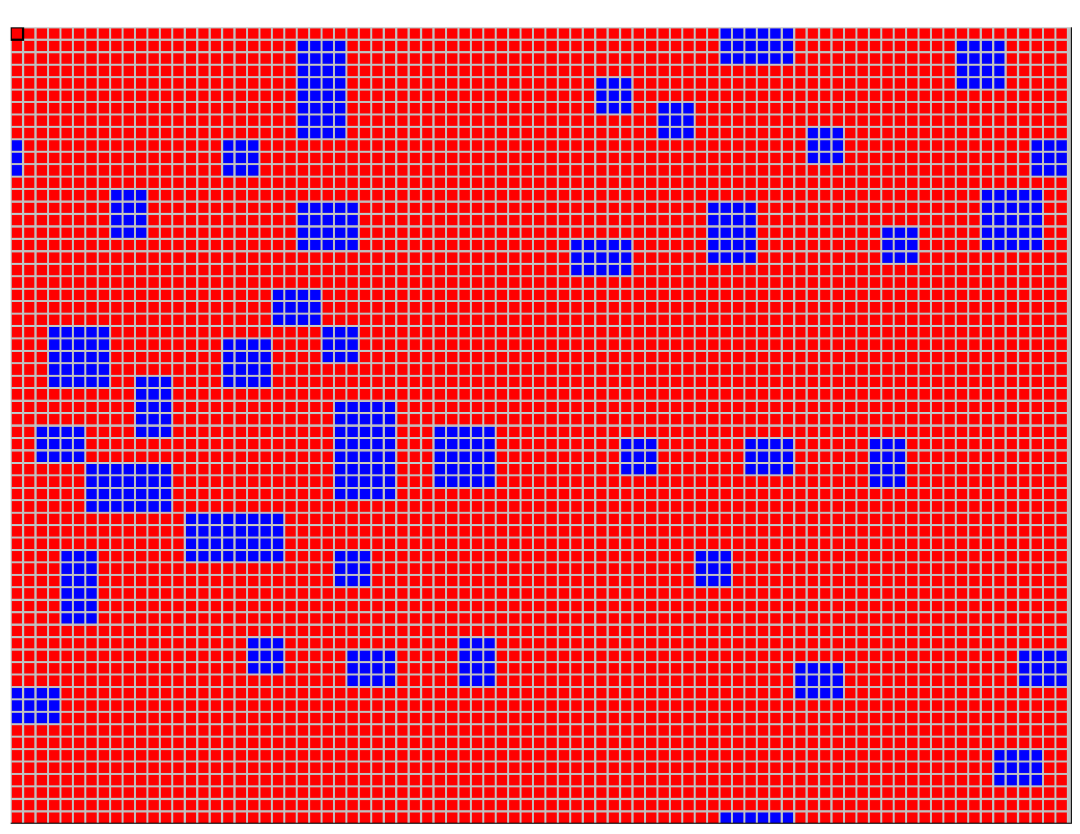

Figure 4: Payoff b=1.70

Discussion

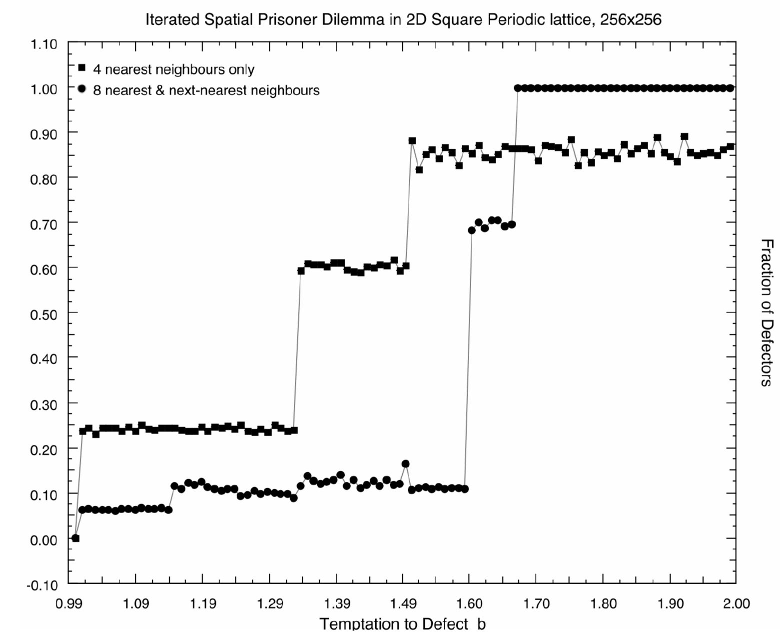

Figure 5: Scan in b showing the phase transition around $b = 1.6666$.

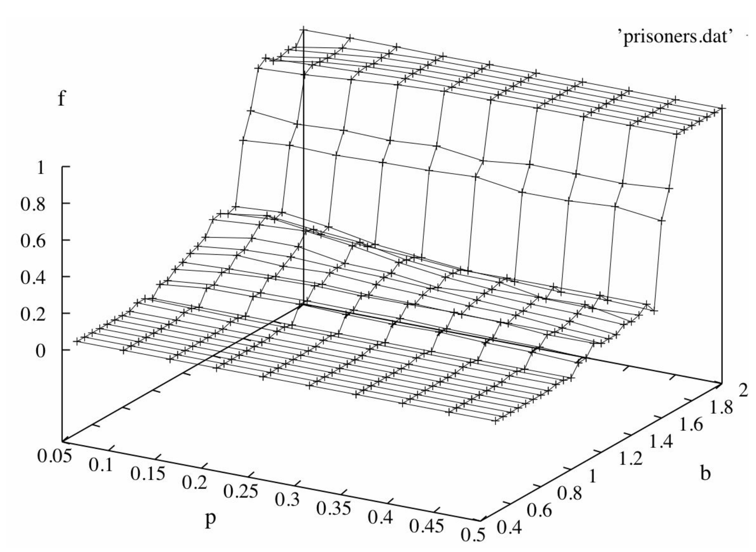

Figure 6: f-p-b surface showing the transition "cliffs."

We have performed a fine-grained scan of the 8-neighbour spatial prisoners' dilemma and by examining individual microscopic configurations have identified the anomalous downward transitions in the fraction f of defectors as defection payoff b was increased. These are explained by oscillations that cease emergence at the downward transitions. Cells that were oscillating between cooperation or defection "resolve their uncertainties" and give rise to the downward edges. The oscillating cases can be for cells around the end of a "rod" or along a sheath around the walls of a "rod". The other upward edges are more easily explained in terms of various conditions of the discrete geometry that support defection as a viable strategy as b increases.

There are a number of variations to the model that remain to be investigated. It is not clear what the relationship between the number of phases and the number of neighbours involved is. Experiments with other spatial geometries and different layers of neighbours may clarify this. It would also be interesting to consider other spatial game strategies other than "always cooperate" and "always defect" for cells to play.

This sort of system can only effectively be studied using a simulation approach. In this case we needed both a highly interactive simulation implementation and a highly optimised fast code capable of making many samples on large systems. Further information on this work is at:

`http://www.massey.ac.nz~kahawick/cstn/037.`

References

[1] M. A. Nowak, A. Sasaki, C. Taylor, and D. Fudenberg, "Emergence of cooperation and evolutionary stability in finite populations," *Nature*, vol. 428, pp. 646–650, April 2004.

[2] E. M. Ronald, M. Sipper, and M. S. Capcarrère, "Testing for Emergence in Artificial Life," in *Advances in Artificial Life: Proc. 5th European Conference on Artificial Life (ECAL'99), Switzerland* (D. Floreano, J.-D. Nicoud, and F. Mondada, eds.), pp. 13–20, Springer-Verlag, September 1999. ISBN 3-540-66452-1.

[3] S. Levy, *Artificial Life The Quest for a New Creation*. Penguin, 1992. ISBN 0-14-023105-6.

[4] K. A. Hawick, H. A. James, and C. J. Scogings, "Roles of rule-priority evolution in animat models," in *Proc. Second Australian Conference on Artificial Life (ACAL 2005)*, (Sydney, Australia), pp. 99–116, December 2005.

[5] M. J. Osborne, *An Introduction to Game Theory*. No. ISBN 0-19-512895-8, Oxford Univ. Press, 2004.

[6] M. A. Nowak and R. M. May, "The spatial dilemmas of evolution," *International Journal of Bifurcation and Chaos*, vol. 3, no. 1, pp. 35–78, 1993.

[7] M. A. Nowak, S. Bonhoeffer, and R. M. May, "More spatial games," *Int. J. Bifurcation and Chaos*, vol. 4, no. 1, pp. 33–56, 1994.

[8] R. Axelrod, "The emergence of cooperation among egoists," *The American Political Science Review*, vol. 75, pp. 306–318, 1981.

[9] A. Rapoport, *Two-Person Game Theory*. Dover Publications, 1966.

[10] C. Hauert and M. Doebell, "Spatial structure often inhibits the evolution of cooperation in the snowdrift game," *Nature*, vol. 428, pp. 643–646, April 2004.

[11] A. V. M. Herz, "Collective phenomena in spatially extended evolutionary games," *Journal of Theoretical Biology*, vol. 169, no. 1, pp. 65–87, 1994.

[12] G. Szabó and C. Töke, "Evolutionary prisoner's dilemma game on a square lattice," *Physical Review E*, vol. 58, pp. 69–73, July 1998.

[13] H. James, K. Hawick, and C. Scogings, "User-friendly scheduling tools for large-scale simulation experiments," in *IEEE Proc. Winter Simulation Conference (WSC07)* (S. H. B.Biller, M.-H. Hsieh, J. Shortle, J. Tew, and R. Barton, eds.), (Washington DC, USA), pp. 610–616, IEEE, December 2007. 1-4244-1306-0.