Visualising the Ginzburg-Landau Field Equation

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Introduction

Many scientific simulations and models are based upon one or more coupled field equations. Fields are often modelled as a regular mesh or grid of individual field variables where each variable is a scalar or vector quantity. Visualising these fields interactively can be a great aid to debugging as well as understanding and interpreting the results of the numerical simulation. Visualisation allows a human observer to inspect a simulation and identify interesting events or phenomena. Visualising scalar field equations is a widely researched area [1], however less research has been conducted with visualising fields of complex numbers or vectors. The visualisation of such fields present extra challenges as each cell has both a real and imaginary part that must be displayed. We discuss the challenges faced with visualising complex fields in two- and three- dimensions and present a number of methods for visualising them as they evolve over time. The need for these visualisation methods arose from our study of the Time-Dependent Ginzburg-Landau(TDGL) equation [2].

Keywords: Time-Dependent Ginzburg-Landau, Vector Field Visualisation

Ginzburg-Landau Theory

Ginzburg-Landau theory provides a framework for modelling the thermodynamic or macroscopic properties of a superconductor without recourse to simulation of microscopic details such as individual atoms or spins. The formulation is based on the concept of a free energy functional for the superconductor that can be written in terms of a complex order parameter, given here as ψ. This order parameter give a measure of how deeply the system is into its superconducting phase. The free energy function has the typical form:

$$F = F_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}\left|(-i\hbar\nabla - 2e\mathbf{A})\psi\right|^2 + \frac{|\mathbf{B}|^2}{2\mu_0} \quad (1)$$

where F_n is the free energy in the normal phase, α and β are phenomenological parameters, m is an effective mass, A is the electromagnetic vector potential, and B (=rot A) is the magnetic induction. Minimizing the free energy with respect to fluctuations in the order parameter and the vector potential give rise to the Ginzburg-Landau equations:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0 \quad (2)$$

$$\mathbf{j} = \frac{2e}{m}Re\left\{\psi^*(-i\hbar\nabla - 2e\mathbf{A})\psi\right\} \quad (3)$$

where j is the electrical current density and Re the real part. Equation 2 specifies the order parameter ψ in terms of the applied magnetic field. This equation has a similar structure to the time-independent Schroeder equation. Equation 3 then determines the superconducting current. These equations suggest that there are two characteristic (physical) lengths that occur in a superconductor. These are usually known as the coherence length.

$$\xi = \sqrt{\frac{\hbar^2}{2m|\alpha|}} \quad (4)$$

which characterises the size of thermodynamic fluctuations in the superconducting phase. The penetration depth λ is given by:

$$\lambda = \sqrt{\frac{m}{4\mu_0 e^2\psi_0^2}} \quad (5)$$

where ψ_0 is the equilibrium value of the order parameter in the absence of an electromagnetic field. The penetration depth then characterises the depth to which an external magnetic field can penetrate the superconductor.

The ratio $\kappa = \lambda/\xi$ is usually known as the Ginzburg-Landau Parameter. Type I superconductors are those for which $\kappa < 1/\sqrt{(2)}$, and Type II superconductors are those for which $\kappa > 1/\sqrt{(2)}$.

In Type I superconductors there is a first order phase transition from the normal state to the superconducting state, and for Type II the transition is second order. This is consistent with Ginzburg-Landau theory.

In 1957, Abrikosov[3] showed that for the case of a Type II superconductor, a (high) magnetic field will penetrate the system in quantized tubes of flux, often in a hexagonal geometric pattern.

Ginzburg-Landau theory also arises as the scaling limit for the XY model and also displays important similarities with the Higgs mechanisms in particle systems.[4].

If we factor the constants and the physical units into scaled variables, we can therefore obtain a simple form of the Ginzburg Landau partial differential equation in term of a dimensionless field u:

$$i\frac{\partial u}{\partial t} + p\frac{\partial^2 u}{\partial x^2} + q|u|^2 u = i\gamma u \quad (6)$$

where the constants $p, q \in \mathbb{C}$ and $\gamma \in \mathbb{R}$.

2D Visualisation

Visualising a complex field like the Ginzburg-Landau equation is more complicated than a scalar field because there is no single value that can be used to represent the state of the cell. Each of the cells within a complex field have a real and imaginary part (which can be transformed into phase and magnitude values). The challenge is to develop a method that can convey both these values to a human observer in an easy to interpret way.

Vector/Arrow Plot

The first method developed to visualise a Ginzburg-Landau field displays each cell as a vector (also known as arrow plots [5]). Instead of the single square of colour often used to visualise scalar fields, each cell is represented by an arrow drawn using a graphics package (See Figure: 1). While this method does display the values of the field, it is not easy to interpret and is only feasible for displaying small fields (the arrows cannot be distinguished in a large field).

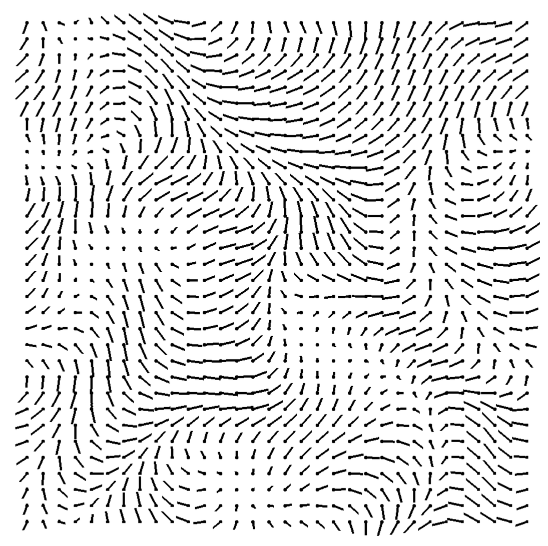

Figure 1: 32x32 TDGL simulation rendered as a vectors.

Phase-Magnitude

The phase-magnitude visualisation method is designed to visualise both the phase and the magnitude of the field in a single display. It renders each cell in the field as a square of colour as defined by the phase and magnitude of the cell. The hue (red-green) is determined by the phase and the brightness (bright-dark) according to magnitude. This method was used to generate the background image from this poster. This method can also be extended by distorting the surface based on the magnitude of the cells. The height of each square's corners is distorted by the magnitude of the four cells it covers. This method is designed to increase the visibility of the magnitude values to allow easier interpretation of the image. This method can be seen in Figure: 2. This method of visualisation can successfully render a complex field in an easy to interpret way without losing information about the phase or magnitude.

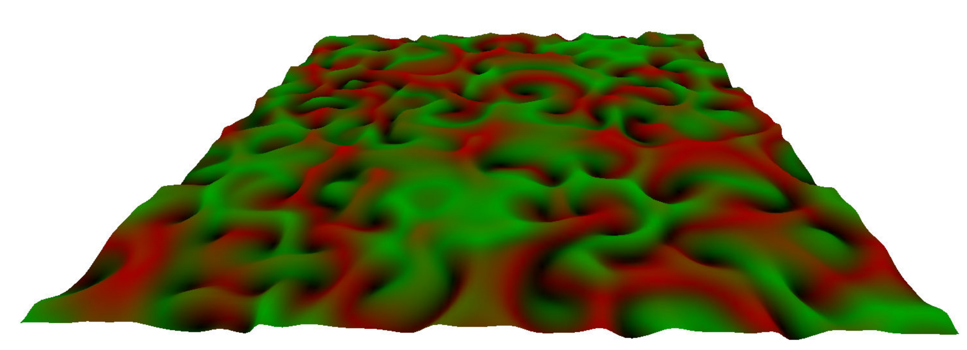

Figure 2: 256x256 TDGL simulation visualised using the Phase-Magnitude Surface Distortion method.

3D Visualisation

Visualising a complex field in three-dimensions is even more challenging than in two. In three-dimensions, visualising the cells as vectors is no longer feasible as the arrows become impossible to see. Instead the field can be rendered with coloured objects as determined from the phase and magnitude values (as with the two-dimensional case).

3D Surface Distortion

The 3D Surface Distortion method renders the outside surfaces of the three-dimensional field in the same way as the two-dimensional phase-magnitude surface distortion method. A field visualised by this method can be seen in Figure: 3. This allows a three-dimensional complex field to be visualised, however it loses all information about the centre of the field.

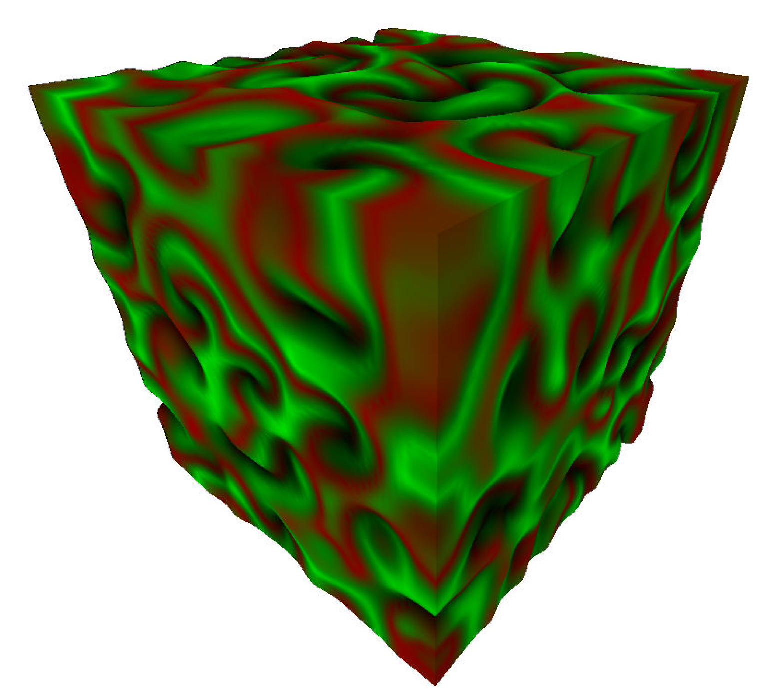

Figure 3: 64x64x64 TDGL simulation rendered with the 3D Surface Distortion method.

Interface Rendering

The interface rendering method was developed to overcome the issue of visualising the internals of a complex three-dimensional field. Instead of attempting to display the all the cells of the field, only the interfaces between domains (as defined by a phase value, in this case 0.3) are visualised. This allows the internal state of the field to be examined by a user controlling the visualisation.

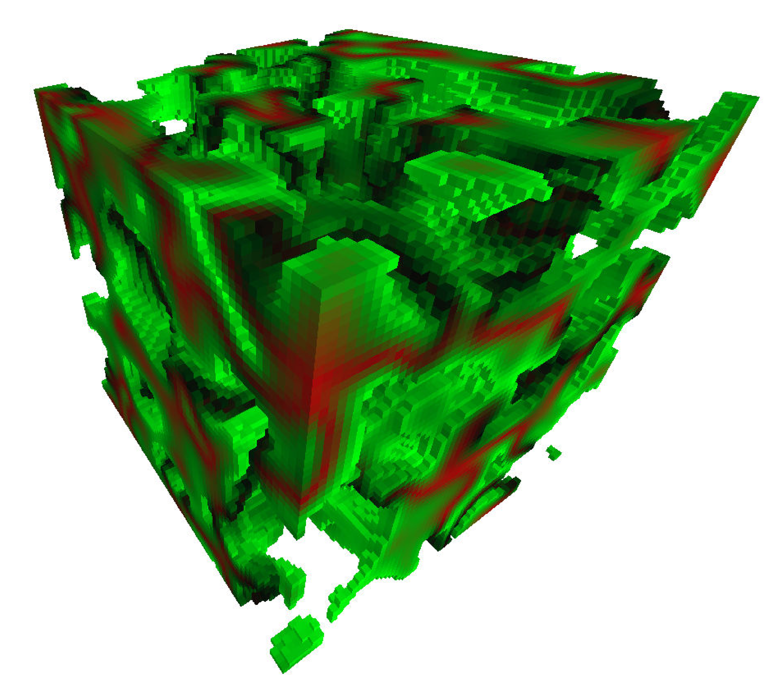

Figure 4: 64x64x64 TDGL simulation visualised with the 3D Interface method.

Conclusions

We have explored a number of ways of representing the real and imaginary parts of the field, in terms of phase ϕ and magnitude $|\psi|$ in terms of colours for scalar potential representations and as explicit arrows to render the vector nature of the field. We have described and presented renderings based on geometrical surface distortions and on interface boundary detection and subsequent colouring. Of particular note was the use of colour and model interface information to allow a cut away rendering of the evolving system, giving important insights into its interior structure and evolution.

For more details see CSTN-074: Visualising Interfaces in Scalar and Vector Field-Model Simulations at www.massey.ac.nz/∼kahawick/cstn/074/

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