Artificial Sand Pictures - A Complex Systems Simulation

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Sand Pictures

Sand pictures are made from a mix of coloured sands and water or oil sandwiched between two sheets of glass and are a common desktop amusement. However, they also provide a good example of mixing and layering in materials science. We construct a lattice-based simulation of a sand picture based around the Kawasaki spin-exchange model with empirical couplings between cells. A Monte Carlo stochastic dynamic scheme is used to update pairs of neighboring cells using a Boltzmann like energy controlled probability process. The sand cells then diffuse around, with a preference parameter for sand to adhere to other sand cells of the same or different types.

This model can be perturbed with a preferred directional gravitational force that leads to nearly correct physical phase separation of the coloured sands. The model provides a visually realistic simulation that can be rendered in real time. We are implementing this using Android and Java on tablet computers with inbuilt gyroscopic sensors that allow the simulated system to adapt to real gravity in interactive time. We describe the model and the implementation and software architecture for this App and the associated performance tradeoffs. We discuss future performance improvements using graphical processing units and other tablet specific features.

Figure 1: Layering in a real sand picture.

The sand picture system is essentially a complex fluid - with a colloidal suspension of sand particles contained in water or oil.

Complex Fluids

Modelling complex fluids is a challenging problem[1, 2], particularly in situations with realistic geometric boundaries and barriers. One useful approach is the Invasion Percolation (IP) model[3] which has developed over many years[4, 5] and is a useful tool for experimenting with flow of immiscible fluids in model reservoir systems.

The IP model has a number of variations in its formulation and has been extensively researched in two and three dimensional configurations. It has been successfully used to model diverse applications ranging from drainage systems[6, 7, 8] to vascular network formation in tumours[9] as well as reservoir extraction and deposition processes where recent applications still find it useful[10].

The conventional IP model is essentially parameterless and has proved a useful tool for exploring the statistical mechanics of percolation as well as a useful model for considering migration within multi phase fluid systems. Hydrocarbons in a reservoir can be considered as occupying the porous volume of particular sort of rock and it is of interest to consider how hydrocarbons – in the form of liquid or gas – will migrate across the rocks and strata etc within a reservoir[11].

The IP model is formulated in terms of an injected wetting fluid that invades the space occupied by a non-wetting defender fluid. The IP model has been investigated for point injection - as might represent a well pipe - and face injection as might arise from inflow from another rock layer. The IP model has a number of statistical features and has been studied for anisotropic cluster effects[12] as well as universality[13, 14]. It has also been used a a basis for comparison with several other statistical models of growth such as: the random field Ising model[15]; the Eden model[16]; Kauffman automata[17]; and the dynamic opinion model[18].

We are interested in the somewhat simpler system of the sand picture in two dimensions. We believe it shares some dynamical growth properties in common with other systems and models that are also driven by an external force such as gravity. Although some research has been reported on the influence of gravity [19, 20, 21], it has not been thoroughly explored as a way of introducing a buoyancy parameter into the IP model.

Implementation

We have developed various simualtion software for models like the Ising, Kawasaki, IP and sand picture system. In this work we are aiming to produce a graphical simulation model that runs effectively in real time so we can watch the complex fluidic behaviour and associated spatial patterns as they form. We have chosen to use C++ as the programming language for both simulation and graphics.

The work shown here relates to the initial prototype Desktop PC implementation. Tablet computers with touch screen displays and built-in gyroscopic sensors allow both detailed user interaction and an automated sense of gravitational direction. We are experimenting with these devices[22] and are presently implementing an App version of the Sand picture simulation models which uses the Java and graphical support system available under the Android Operating system for mobile devices.

The Model

We base our sand model on a lattice of cells which contains a single variable that determines whether the cell is the suspending "water" or "oil" or contains a sand particle. We can further refine the model with multiple sand species or colours, which can be heavier or lighter.

We need to impose a dynamic scheme to determine how sand and the suspending liquid move around. A useful starting point for this is the Ising model on a lattice. In the Ising model, a heat-bath algorithm is used to emulate thermal effects on atoms in a magnetic material arranged in a crystalline lattice. The Ising system consists of a micro crystalline array of single bit magnetic moments or "spins" which interacts with its nearest neighbours. At each time step of the simulation each spin is considered in turn and the energy and thermal probability of it "flipping" – reversing its direction are considered. The probability of flipping is different, depending upon the applied temperature.

Ising spins align with their neighbours when the system is cold, but thermally randomize when it is hot. The interesting feature about the Ising system in 2 (or 3) dimensions is that there is a definite Curie temperature that can be measured. In real magnets the Curie temperature is the temperature above which the material stops being a magnet, or an alternative viewpoint is that materials like iron spontaneously become magnetic below their Curie temperature. This is known as a phase transition and is very difficult to explain simply without a model to demonstrate.

In the sand system however we need to fix the number of sand and suspending liquid cells. A useful related model is therefore the Kawasaki exchange model which is constructed in a similar manner to the Ising system. In this case however we preserve a fixed ratio of the two microscopic species since instead of flipping or changing species, in the Kawasaki system we only allow them to swap positions with one of their (randomly chosen) neighbours. In this respect the Kawasaki system models diffusion and phase separation or "unmixing" of the two species. The rate and manner of unmixing is like the separation of two atomic species in a binary alloy. This sort of dynamical behaviour is of great importance in real materials. Without some separated granules an alloy typically lacks strength and other physical properties but if too much separation occurs it can break apart and cause catastrophic failure in for example fuel rods in a reactor. In the sand system, an exchange or cell-switching dynamical scheme similar to the Kawasaki model can be used.

We have already experimented with a variation of the Kawasaki model in 2-D and 3-D with a gravitational bias imposed[23]. It successfully exhibits complex layers with multiple phases. In this present work, we experiment with tuning the microscopic rules to try to obtain a more realistic set of behaviours for the sand picture.

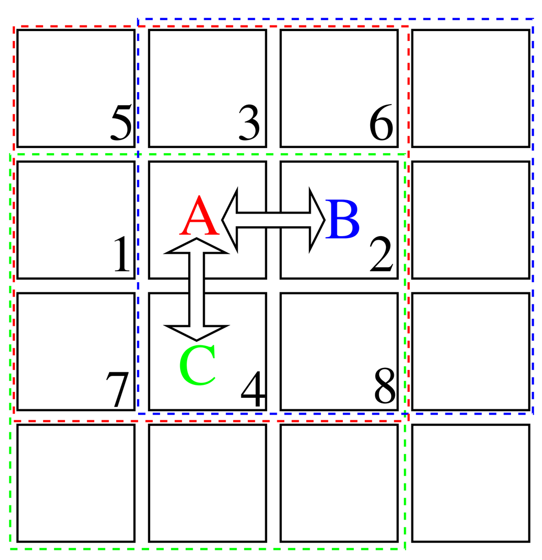

Figure 2: Mesh exchange dynamical schematic.

Figure 2 shows a 2-dimensional mesh surrounding individual cells A (red), B (blue) and C (green). The colours show the surrounding neighbourhood "haloes" for these cells. We can use nearest-neighbour or Moore neighbourhoods with 4 (1,2,3,4) or 8 (1,2,3,4,5,6,7,8) neighbouring cells respectively. For the exchange or switching dynamics, we consider the energy consequences of particle A switching with particle B or with particle C. In the case of A and B they are both at the same height and therefore gravity playes no part - unless the system is at a angular tilt as shown in the screenshots in column 3. The case of A and C will have gravitational consequences and there should be a bias so that the heavier of A and C prefers to move downwards.

To make the model behave realistically the Boltzmann approach allows a heavier particle to move upwards - but only with a very low probability. This stochastic dynamics emulates diffusion amongst the particles and also compensates for the rigidity of the mesh.

The simulation algorithm therefore consists of the following stages:

- Initialize a mesh of for example 1024 by 768 cells, that can be mapped to individual pixels on a display.
- Populate each cell randomly with either water or heavy sand or light sand.
- Iterate time steps of the model where each step consists of:
  - Consider each cell in random order
  - Look at the cell "below" the chosen cell
  - Follow the microscopic rules given in column 4 to determine whether to exchange or switch the contents of the two cells
  - Repeat

This model is essentially a variant of the Kawasaki Spin exchange model, where we have taken the Boltzmann energy probabilistic rate equation usually used in a Metropolis update scheme and made it more sophisticated by adding additional terms and effects to take account of gravity and a pseudo-viscosity within the complete fluid suspension.

Simulated Snapshots

This series of screenshots shows the simulated sand system rendered with the Desktop PC version and showing the orientation of the system with respect to the gravitational direction, on the right hand side. In each picture, the system consists of a suspending liquid (water or oil) represented by a blue particle species; a light yellow coloured sand species and a heavier black coloured sand species.

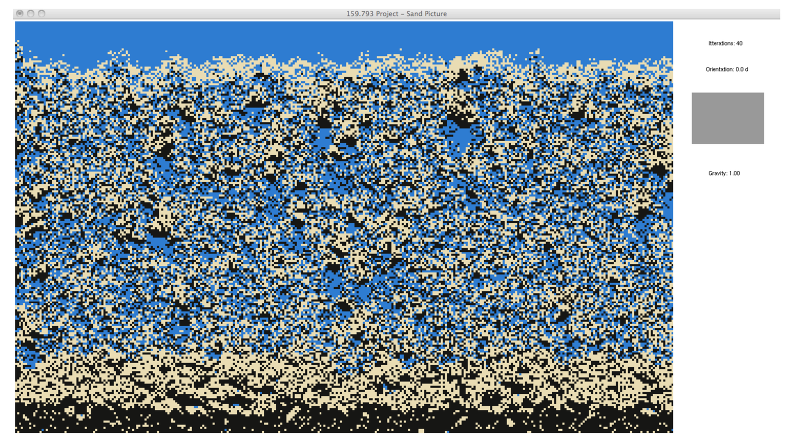

Figure 3: Sand starting to layer

Figure 2 shows the Desktop PC version of the sand simulation after 40 time steps. Following a random initialization, after only 40 time steps the black heavy particles are already starting to fall the fastest and are starting to accumulate at the bottom.

Figure 4: Layered sand simulation

Figure 3 shows a simulation at its later stages when the particles have fallen into layers with pockets of lighter material spread throughout. This shows a similar layer behaviour to the actual sand picture.

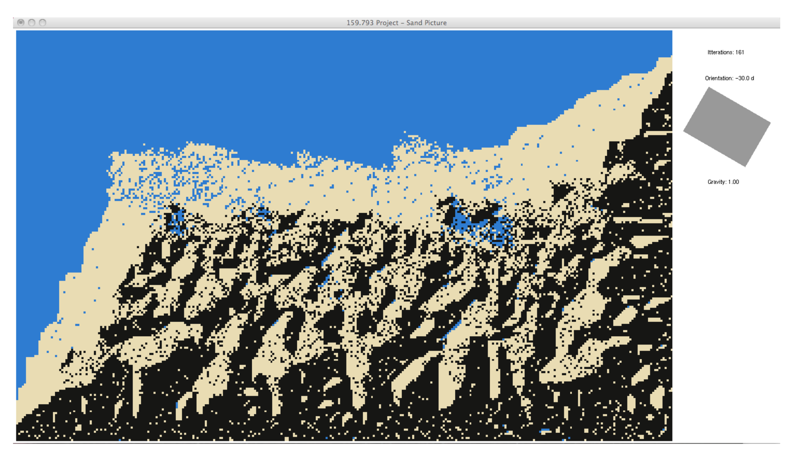

Figure 5: Layered sand with rotation changed

Figure 4 demonstrates the ability to rotate the sand picture and the layering effects it has on the sand simulation.

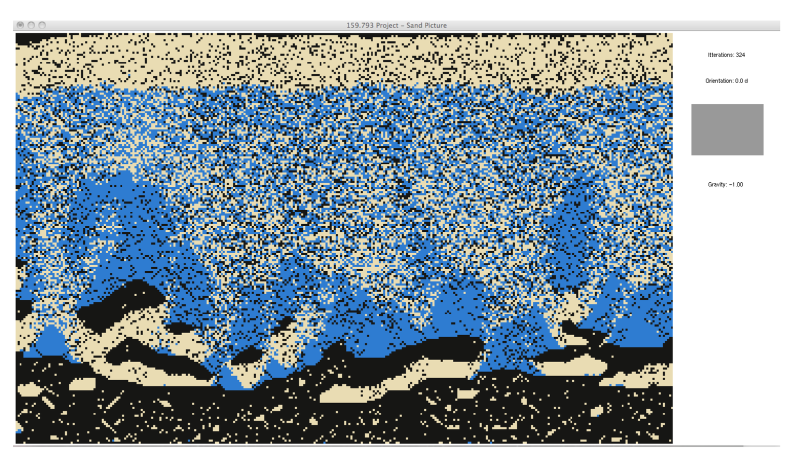

Figure 6: Relayering after change in gravity

Figure 5 shows the effect and re-layering after a gravity change in the sand simulation. As can be seen the lightest water particles are falling to the bottom and the heaviest black sand are moving to settle at the top of the screen.

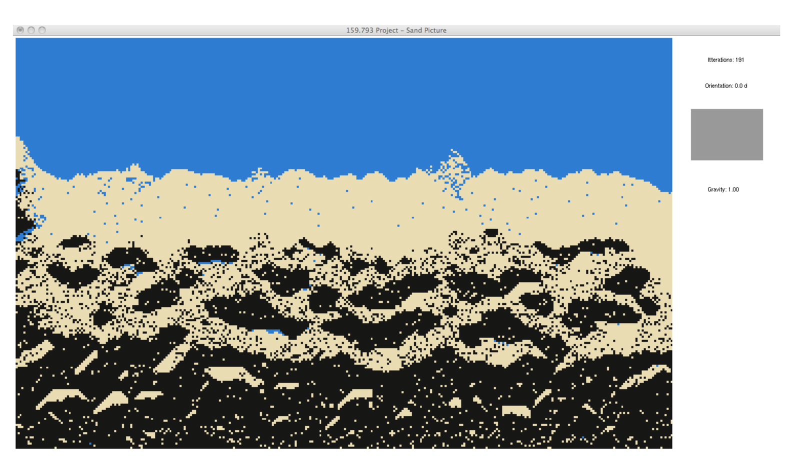

Figure 7: A near steady state of the simulation

Figure 6 shows a nearly steady-state of the sand simulation. This shows the effect that would happen in real life it the sand picture has been left for a few minutes for the particles to rearrange themselves.

Empirical Rules

There are a number of parameters that need to be investigated experimentally to tune the macroscopic behaviour of the system. The relative strength of gravity compared to the normal random diffusion of particles is one. Another is the particular neighbour set or threshold to consider to introduce a realistic looking pseudo-viscosity.

We arrived at a rule set rule set for the evolution from one state to another by experimentation. Through testing of various rulesets, the following rules gave the most realistic implementation

The simulations acts on the particle that that is defined be adding to the x and y coordinates of the current position. The values that are added to the x and y are defined in a look-up table. This is modified by the orientation of the system, on the Desktop PC version the state of both the gravity and the user defined rotation modify the look-up table. This means the rules can be defined arbitratly, it also means that the rules do not have to be heavily modified for the port to mobile devices which use a combination of Android/Java.

The rules make the use of random number generation for tie=breaks and for some of the stochastic conditions, and are as follows:

- If the sand is white (lightest sand)
  - if there are seven or more white sand particles around the white sand particle then do nothing
  - if the same white sand particle or the heavier black sand is "below" then half the time look and see if there is water particles (lightest) to the bottom left and right otherwise do nothing
  - if both are water to the left and right then coin toss to see which one we swap with. Otherwise fall into the one that as the water.
  - if no water present then do nothing
- If the sand is black (heaviest sand)
  - if there are five or more black sand particles around the particle then do nothing
  - if the same black sand is below then try to fall to the left or right of the if there is a light sand or water is on either side
  - if the particle below is water then switch the particles
  - if the particle below is white sand
    - 1/3 of the time swap the heaver sand for the lighter sand
    - 1/3 of the time see if there is water below left and right if there is then push the white sand left or right and let the black sand drop down
    - the final 1/3 of the time if there is a lighter particle below to the left or right then fall into its place (swap)

These rules conserve the amount of each particle to that which was first initialized in the system. This means we do not lose any particles from the simulation. In a real life sand picture the mounting case that holds the sands and water is sealed so it is also impossible to lose or change the proportion of particles present. Real life systems often leak air bubbles into the system and an area for future work is to explore how air bubbles might be suitably simulated.

Other areas for further investigation concern the dimensionality of the system. A real sand picture is almost two-dimensional, with a very thin region of colloidal fluid between the glass plates. It is an open question as to how this thickness in the third dimension will affect the complex fluid flow. A simulated system may be able to probe this issue.

See: http://www.massey.ac.nz/~kahawick/cstn/196/cstn-196.html for more information.

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