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Emergent System Effects from Microscopic Evasion Choices in a Predator-Prey Simulation

C. J. Scogings and K. A. Hawick

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A wide range of predator evasion strategies have been reported for several real predator prey systems in the wild. We investigate predator evasion in system of many simulated animal agents. Our model is capable of simulating the emergent effects arising from around a million individual microscopic agents which make individual intelligent choices based on their local information. We find oscillatory and other system wide effects arising from enhanced abilities of prey to evade their predators. We compare some of these effects to real predator-prey observed patterns of behaviour. We find additional oscillatory effects arising when prey can evolve towards different levels of evasive behaviour.

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Prof Ken Hawick, Computer Science, Massey University, Albany, North Shore 102-904, Auckland, New Zealand. Complete List available at: <http://www.massey.ac.nz/~kahawick/cstn>

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Chris J. Scogings
 Computer Science
 Massey University - Albany Campus
 North Shore 102-904, Auckland
 New Zealand
 email: c.scogings@massey.ac.nz

Ken A. Hawick
 Computer Science
 Massey University - Albany Campus
 North Shore 102-904, Auckland
 New Zealand
 email: k.a.hawick@massey.ac.nz

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ABSTRACT

A wide range of predator evasion strategies have been reported for several real predator-prey systems in the wild. We investigate predator evasion in a system of many simulated animal agents. Our model is capable of simulating the emergent effects arising from around a million individual microscopic agents which make individual intelligent choices based on their local information. We find oscillatory and other system wide effects arising from enhanced abilities of prey to evade their predators. We compare some of these effects to real predator-prey observed patterns of behaviour. We find additional oscillatory effects arising when prey can evolve towards different levels of evasive behaviour.

KEY WORDS

agent-based model; animat; overcrowding; prey evasion; evolved behaviour; intelligent agents.

1 Introduction

Predator-prey systems are often identified in nature and some of the effects understood from direct observation. Many of the system wide emergent effects are however still poorly understood, but can be probed using computer simulations. In this paper we investigate emergent oscillatory phenomena arising from population fluctuations when prey are able to evade predators.

Agent-based “Artificial life” models that simulate predator-prey systems are well known [1–5]. Such models consisted of agents that were entirely virtual and did not attempt to model real animals. Further work moved towards a modicum of real animal behaviour leading to the term “animats” [6, 7] being used for agents that attempt to model some aspect of real animal behaviour including flocking [8], sentinels [9] and ter-

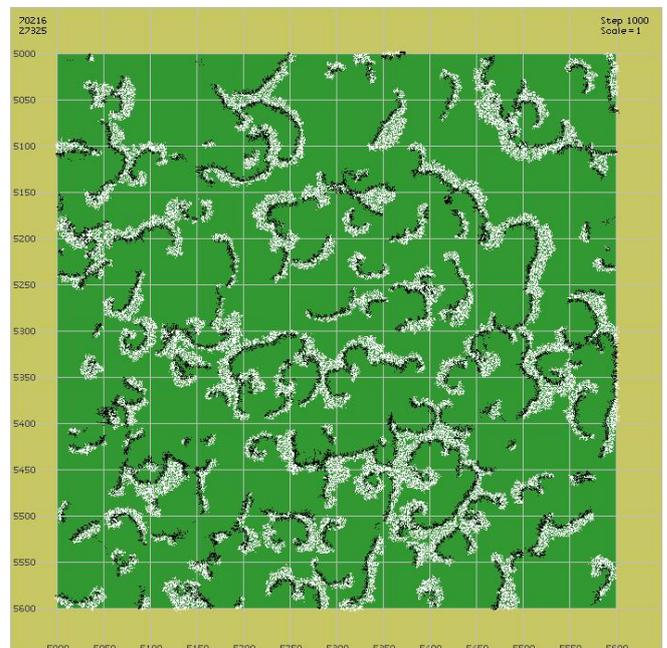


Figure 1: The situation at step 900 of a typical run showing animats on a square grassed area. Predators are black and prey are white. Various macro-clusters, including spiral formations, have emerged.

mites [10].

Many agent-based models focus on “emergence” – the complex and often unexplained patterns and clusters that emerge from the interactions of many agents at the local level. In predator-prey models, emergence can take the form of the defensive spirals and other features discussed in [11] and shown in Figure 1.

One aspect of real predator-prey behaviour is the use of evasion techniques by prey [12–14] and how such evasion ca-

pabilities affect both predators and prey. Of particular interest to this investigation is that large numbers of prey can lead to overcrowding which reduces the ability of individual prey to evade predators. Examples include: predators that employ a “sit-and-wait” technique will usually benefit from overcrowded prey [15]; prey that rely on hiding in burrows can have hiding places reduced by overcrowding [16]; and antelope in large herds are less vigilant than those in smaller groups [17].

As the foundation for an investigation into prey evasion, this paper makes use of a well-established spatial predator-prey model [18]. The model consists of a flat plain on which large numbers of animats reproduce, feed, flee predators or hunt prey and eventually die. The model reproduces the repetitive cycle of behaviour that is well known from predator-prey equation-based models such as the Lotka-Volterra equations [19]. Predators kill prey and, if prey is plentiful, the predator numbers increase, leading to an increased demand for prey which causes the prey population to drop and this in turn causes a drop in the predator population which allows the prey population to recover, and so the cycle continues.

The model abstracts over the exact mechanisms for traits such as prey evasion to evolve or appear. The animat model we use has a large enough population that we can study such effects from a statistical perspective over several generations. Observations are therefore insensitive to microscopic implementation details on how such behaviour would actually be passed on in real biological systems. We present results on introducing: different (but fixed) evasion abilities; the effect of reducing evasion abilities when prey becomes overcrowded; and the evolution of evasion abilities over several generations.

In this paper we investigate the effect on both predator and prey populations, and thus on the carrying capacity of the system as a whole, when prey make use of evasion techniques. A brief overview of the predator-prey model is provided in Section 2. The introduction of a fixed and identical evasion ability for all predators is discussed in Section 3. Section 4 investigates the effects of a reduction in prey evasion ability due to prey overcrowding. Section 5 allows prey to evolve the evasion ability through mutation across generations and investigates how the evolutionary process affects both the predator and prey populations. We discuss the implications of traits such as evasion on the overall system in Section 6 and offer some conclusions and ideas for future work in Section 7.

2 The Predator-Prey Animat Model

The model contains two groups of interacting agents (or “animats”) – the predators and the prey. Every animat maintains its current state including: current health; current age; and an x-y location on the flat, 2-dimensional map. Every animat also carries a set of rules (depending on species). The rule sets are listed in Table 1.

Table 1: Animat Rule Sets in Priority Order

Rules for predator animats:	Rules for prey animats:
1. breed if health > 50% and mate adjacent	1. flee from predator if predator is adjacent
2. eat prey if health < 50% and prey adjacent	2. graze (eat grass) if health < 50%
3. seek mate if health > 50%	3. breed if health > 50% and mate adjacent
4. seek prey if health < 50%	4. seek mate if health > 50%
5. randomly move to any adjacent position	5. randomly move to any adjacent position

Each animat is initialised with the current age set to zero. The age is incremented at every time step of the simulation and when it reaches a pre-set maximum the animat “dies of old age” and is removed. When a new animat is produced, its current health value is set to the health of its parent. From then on, the current health is reduced at each time step and if it reaches zero the animat “starves to death”. If an animat eats then the current health value is increased by a certain amount. The concepts of health values and animats eating behaviours are discussed in [20].

Prey eat “grass” which is placed at specific locations on the map – usually in a contiguous area. Grass has a fixed “nutritional value” and this is the number of health points that prey receive when executing the graze rule. In these experiments grass has a value of 45. Thus if a prey animat has 5% health and executes the graze rule, the animat’s health would increase to 50%. However, if a prey animat with 75% health executed the graze rule the animat’s health would rise to 100% as current health may not exceed 100%. The experiments discussed in this paper are situated on a large square “grassed area” which explains why the diagrams showing animat locations have a distinct edge. Containing the animats is useful as it prevents populations becoming unmanageable and also limits the area of the (otherwise unbounded) grid in which the animats exist. Previous work [21] has shown that these limitations do not affect the emergent patterns and clusters of the model.

Predators eat prey but only do so if the predator is “hungry” (i.e. the current health is less than 50%) and the prey is adjacent to the predator. Early on in the development of the model, a problem was identified whereby several predators simultaneously consumed the same prey animat. This led to the situation where a large number of predators could be sustained by an unrealistically small number of prey. This problem was solved by immediately removing “consumed” prey from the list of available animats in the given time step.

Animats are updated in a random order which removes any spatial artifacts from the sweep order. The process is thus a two-phase system in which the variables for all animats are updated after all rules have been executed. A full discussion of this (and other) methods of updating agent-based models can be found in [22].

Rules are considered in a strict priority order. Each time-step, every animat attempts to execute the first rule in its rule set. However, most rules have conditions and often cannot be executed. For example, predators can only eat prey if prey are adjacent. If the conditions for the first rule can not be satisfied, the animat attempts to execute the next rule in the set and so on. Breeding only has a certain chance of success. This is a simple alternative to factoring in a host of complicated parameters including birth defects, nutrition, adequate shelter and so on. Changing the chances of a successful birth can dramatically alter the number of animats and can sometimes cause the extinction of all animats. For these experiments the chance of a successful birth was set to 15% for predators and 80% for prey.

3 Experiment 1 – Evasion Values

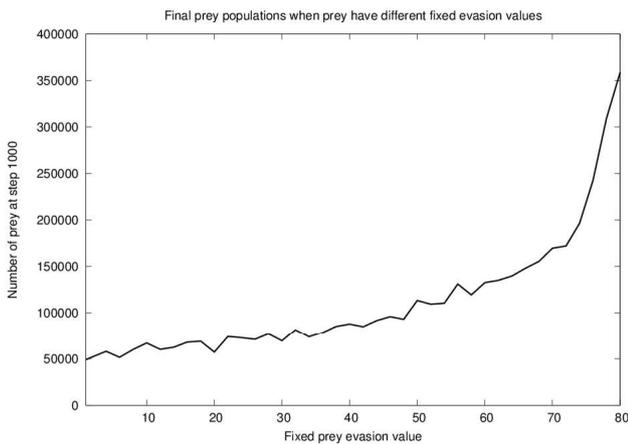


Figure 2: Plot showing the effects of fixed prey evasion values on prey populations. As evasion becomes more effective, predators catch less prey allowing the prey population to increase.

In this experiment, prey evasion of predators was introduced into the model in the following way: each prey agent was assigned an “evasion value” which is an integer value in the range 0 to 99. The evasion value is the percentage chance that the prey will evade a predator. Hence, for example, there is a 10% chance that prey will evade a predator when the prey animat has an evasion level of 10 and there is an 80% chance that prey will evade a predator when the prey animat has an evasion level of 80, and so on.

Prey evasion is checked during the execution of a predator’s “eat” rule. When a predator is adjacent to prey and executes the “eat” rule, the chance of prey evasion is calculated. If the prey successfully evades the predator, the predator does not eat and thus does not receive the increased health from eating.

At the start of any simulation, all prey receive the same initial evasion value. The model can then be run in one of two ways: either every prey animat always carries the initial evasion value, i.e. all prey are clones of the initial prey; or the evasion level is allowed to mutate and evolve from one generation to the next.

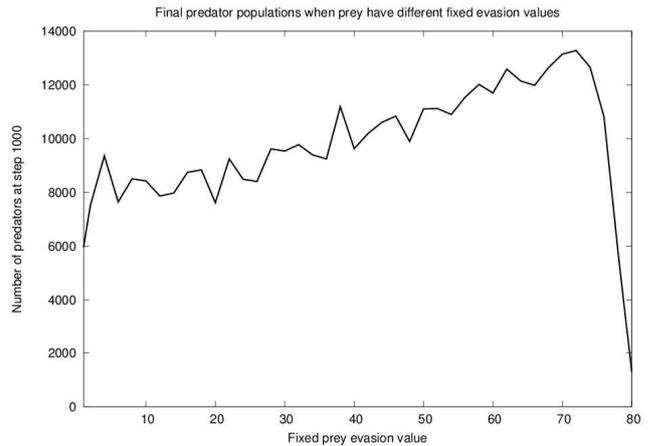


Figure 3: Plot showing the effects of fixed prey evasion values on predator populations. As evasion becomes more effective, predators catch less prey causing the prey population to increase and in turn making it easier for predators to find more prey. The predator population thus also increases – at least initially.

In this experiment, only the initial evasion values were used (all prey were identical clones). This enabled an analysis of the effect on both prey and predator populations of an evasion value that was fixed across the prey population. Several fixed evasion values were tested and the results are shown in Figure 2 (prey population) and Figure 3 (predator population). Each data point in the graphs is the final population figure at the end of a simulation of 1,000 time steps during which all prey carried the designated evasion value.

There is a distinct change in the emerging formations of animats during these experiments. Figure 4 shows the situation during a run where all prey have a fixed evasion value of 60. Because prey now have considerable success in evading predators, the animat clusters have taken on a more diffuse nature with prey in larger clusters. There are also small regions where predators have died out and only prey remain. This situation should be compared with Figure 1 in which prey have an evasion value of zero.

An unexpected outcome of this experiment is that the preda-

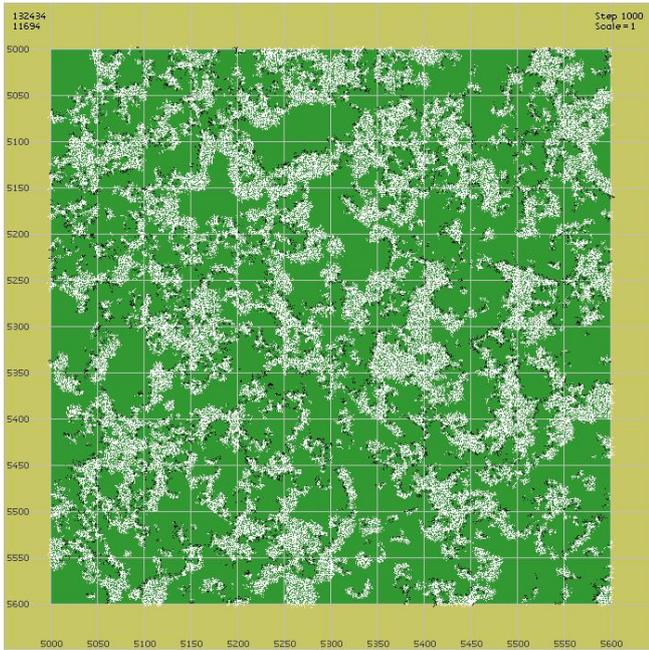


Figure 4: The situation at step 1000 of a run in which all prey have a fixed evasion value of 60. Predators are black and prey are white. The tight formations shown in Figure 1 have disappeared and animats are spread in a far more diffuse manner. This screen shot contains 11,694 predators and 132,434 prey animats.

tor population initially benefits from increasing the prey evasion value, even though increased evasion values mean that individual predators catch less prey. The reason for this is that the increased evasion chance allows individual prey to escape and breed, thus increasing the prey population. This increased prey population, in turn, provides predators with more available prey and thus the predator population also increases, following the well known boom-bust population phenomena [23].

However, when the prey evasion value reaches values above 70, individual prey becomes too difficult to catch and predators can not catch enough to sustain themselves. Thus, from this point on, there is a rapid downward trend in predator numbers and a corresponding increase in prey population.

4 Experiment 2 – Overcrowding

Experiment 1 in section 3 established that both predators and prey benefit from prey evasion up to the point when the prey evasion value moves above 70. Once the value is greater than 70, prey can successfully evade predators most of the time leading to predators starving and the predator population becoming unsustainable.

In real animal populations, prey evasion does not reach such

levels as some other factor usually intervenes to prevent this. These may include: predators evolving speed or other attributes that reduce prey evasion effectiveness, predators switching to other prey, or overcrowding of prey leading to a reduction in the ability to evade predators [15, 17, 24].

In this experiment, we assume that when the prey population exceeds 50,000 the prey will become overcrowded and that this overcrowding leads to a reduction in the prey evasion value. At this stage, the reduction is uniform across the prey population but future work may include making the reduction dependent on local factors. The formula used to calculate the reduction is (where N is the total prey population):

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if (N > 50,000) then:
    reduction = (N - 50,000) / 1,800
else:
    reduction = 0

```

The reduction is then applied to the evasion value for each prey animat.

Once again, all prey were assigned a fixed prey evasion value and offspring were clones, i.e. every prey animat only ever carries the initial fixed evasion value. Several fixed evasion values were tested and the results are shown in Figure 5 (prey population) and Figure 6 (predator population). Each data point in the graphs is the final population figure at the end of a simulation of 1,000 time steps during which all prey carried the designated evasion value.

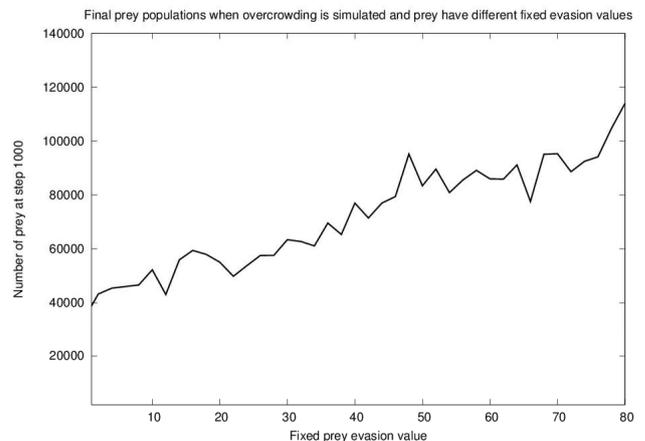


Figure 5: Plot showing the effects of fixed prey evasion values on prey populations. Predators are assumed to benefit from prey overcrowding and prey evasion values are reduced if the population is over 50,000. As evasion becomes more effective, predators catch less prey causing the prey population to increase.

The effects of prey overcrowding are clearly shown in Figure 5 where the prey populations never climb above 120,000. This should be compared with the situation in which prey

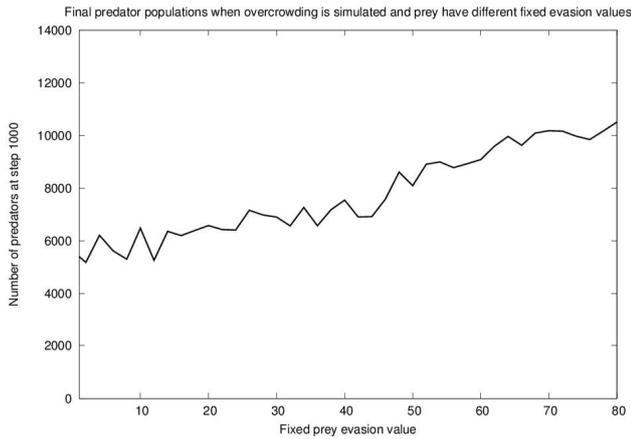


Figure 6: Plot showing the effects of fixed prey evasion values on predator populations. Predators are assumed to benefit from prey overcrowding and prey evasion values are reduced if the population is over 50,000. As evasion becomes more effective, predators catch less prey causing the prey population to increase and in turn making it easier for predators to find more prey. The predator population thus also increases.

overcrowding was ignored and prey populations reached figures in excess of 350,000 – see Figure 2. Figure 6 shows the benefits to the predator population of prey overcrowding in that predators are always able to catch enough to eat and to increase their population, even when prey evasion values are above 70.

5 Experiment 3 – Evolution of Evasion

Experiment 1 in section 3 established that different (fixed) prey evasion values affected both the predator and prey populations. In particular if the evasion value was too high (above 70) it became impossible for predators to sustain themselves and the prey population consequently increased dramatically. Experiment 2 in section 4 showed that both predator and prey populations could be stabilized by introducing a reduction in prey evasion values based on the size of the prey population. This experiment investigates what happens when prey are allowed to evolve an evasion value. In this experiment, all prey were initially assigned an evasion value of 0 but evasion values were allowed to evolve due to mutation. When a new prey animat is produced, it inherits the evasion value of its parent but makes a random change to the evasion value (mutation). This change can be as much as 5 more or less than the inherited evasion value. For example, if an existing prey animat has an evasion value of 25 its offspring can have evasion values anywhere in the range from 20 up to 30. However, prey evasion values are restricted to a minimum of 0 and a maximum of 99.

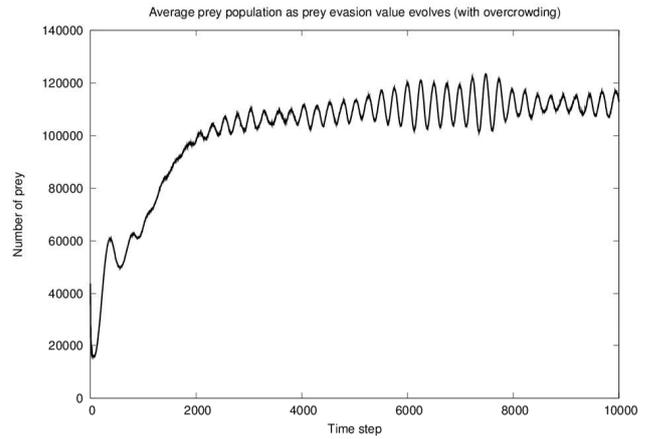


Figure 7: Plot showing the effect on the prey population as prey evasion values are allowed to evolve naturally over time. All prey were initially assigned an evasion value of 0. The overcrowding reduction is applied to evasion values, as described in Experiment 2 in section 4. The plot shows the average population over ten runs with different random number seeds.

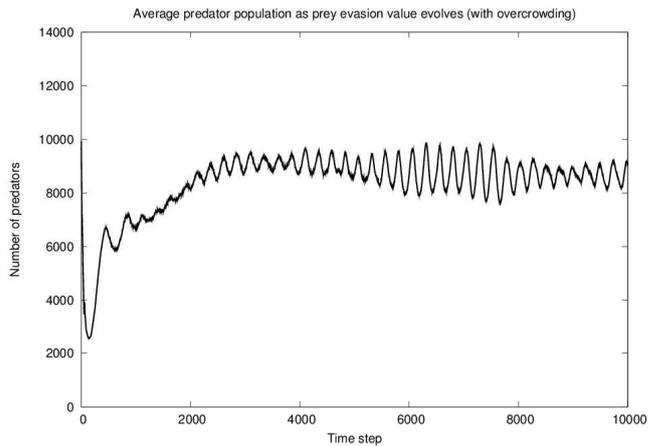


Figure 8: Plot showing the effect on the predator population as prey evasion values are allowed to evolve naturally over time. All prey were initially assigned an evasion value of 0. The overcrowding reduction is applied to evasion values, as described in Experiment 2 in section 4. The plot shows the average population over ten runs with different random number seeds.

Figure 7 and Figure 8 show the average populations of prey and predators respectively over time as the evasion values mutate and evolve within the prey population. Evasion values initially climb rapidly as individual prey mutate to a higher evasion value, enabling them to more easily evade predators and thereby survive to produce more offspring. However, the prey population increases to the point where prey become

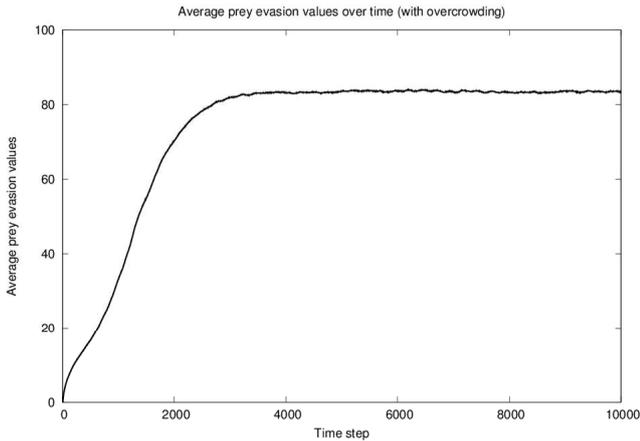


Figure 9: Plot showing how the prey evasion values evolved over the simulations shown in Figure 7 and Figure 8.

overcrowded and a reduction is applied to the evasion values. This enables predators to more easily catch prey and thus stabilizes the populations in the typical “boom-bust” oscillations characteristic of predator-prey models [23].

Figure 9 shows the change in the average prey evasion value over time. There is an initial increase followed by a steady state as the populations stabilize. The curve is quite smooth, albeit with continued fluctuations superposed as animats adjust their evasiveness locally.

6 Discussion

An interesting and important general observation concerns the interplay between what strategies work well for individual agents and what works best for the system as a whole. As figures 2 and 3 show, a prey species that is good at predator evasion may well survive initially but this can lead to massive over population levels of prey which in turn lead to a massive increase in predators before the whole system crashes. Predators then wipe out all prey before dying themselves of subsequent starvation. In many other variations of the core model we have found that such crashes are often avoided and the spatial model “equilibrates” and settles to finite dynamic mean population values around which stable and regular oscillations occur.

As the experiments in section 4 showed, a more realistic prey evasion probability does allow the system to reach a dynamic equilibrium mean values without complete extinction crashes occurring. In general if the spatial model system is large enough, even modestly large patches of local extinction can be tolerated and the system as a whole will recover. There are no obvious systematics to the fluctuations in the plots of section 4.

The most interesting case is when individual animats are able

to adjust their microscopic behaviours and we observe a more sophisticated set of oscillations present. In addition to individual predator-prey coupled oscillations with a period of approximately 270 time steps, another slower envelope of 1,350 time steps emerges. We hypothesise that this is the adjustment time constant for interacting spatial regions of interacting animats to appropriately mix and adjust to prevailing conditions.

There is another very much slower envelope observed in the population plots, with a period in excess of 5,000 time steps and that we believe from prior work is related to the size of the model box region. The oscillation is likely caused by reflections from the boundary. A larger system size would have longer time constant, or by imposing periodic boundary conditions the effect could likely be removed. It is not overly influential for this study of evasion effects however. Figure 4 suggests that at any given time there are a number of relatively localised subsystems of interacting predators and prey in the model system. If a local extinction does occur, then stragglers colonize and take over that empty region and so the overall system - providing it is large enough - will not suffer total extinction, due to local fluctuations. A systemic effect such as massive over success of, for example, prey (as found in Experiment 1 in section 3) is necessary for a complete crash.

There is scope for a more detailed Fourier time-series analysis [25] averaged over sample configurations to see if relationships between these periods and the model parameters can be found.

Real animals observed in the wild likely interact over a highly localised region. Nevertheless the individual regions - pride groups and so forth - will still interact at their regional boundaries and hence individual choices will have an effect on the system as a whole. The time constant for this coupling together of localised regions is an interesting area for further investigation and could have implications for game management decision making - animal relocation; fence and boundary management and related practical options.

7 Conclusion

We have shown how predator evasion can be incorporated into an individual agent-based “animat” model to produce a number of emergent effects on the system as a whole. These include: over population of prey followed by over predation and subsequent system wide population crashes; and longer-term approach to a dynamic mean equilibrium around which stable spatial fluctuations are possible.

We have also observed the superposition of oscillations which we hypothesize is due to a new regional effect caused by local adjustments by individual animats within a region. We have shown how our rule-based model supports individual animats

evolving traits and effectively adjusting a model parameter to reach a dynamic equilibrium that is stable against whole system crashes.

We believe there is ample scope for a more extensive quantitative analysis to further investigate interactions between localised regions. It may also be possible to investigate an imposition of deliberate boundary walls or isolation artifacts such as fences and animal group relocations, both in the model as well as in real predator-prey systems.

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