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Non-Monotonic Phase Transition Edges in the Spatial Prisoners' Dilemma

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Technical Report CSTN-037

Non-Monotonic Phase Transition Edges in the Spatial Prisoners' Dilemma

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ABSTRACT
Simulations of complex systems models provide a numerical laboratory for studying emergent spatial phenomena. We report on simulations of the 2-dimensional spatial prisoners' dilemma with both 4- and 8-neighbour playoffs. We identify non-monotonic dependence of the convergent population of defectors with the variation of defection temptation reward in the 8-neighbour system. We present some microscopic configuration analyses to explain this phase transitional effect.

KEY WORDS
prisoner dilemma; game theory; agent population dynamics; simulation.

1 Introduction

Simulation has an important role to play in the study of emergence in complex systems. A problem of continued interest is the way that both cooperation and spatial order emerge from many component systems [11]. The spontaneous emergence [14] of spatial structure is of continued interest in a variety of complex systems including artificial life models [4, 7]. Spatial emergence can be modeled using a game theoretic formulation [12] such as the iterated spatial prisoner dilemma [9, 10]. This scenario is based on the well-known two-player prisoner dilemma [1]. Two prisoners accused of being partners in a crime are interrogated separately and are separately given the options to stay silent (cooperate with one another) or to testify against their partner (defect). If both cooperate they both go free and each receive the payoff reward R. If the other player cooperates then defecting carries the temptation to defect payoff of T. The sucker payoff for cooperating when the other player defects, is S.

The usual definition for the prisoner's dilemma is for:

$$T > R > P > S \tag{1}$$

This can be rescaled so that $T = b$, $R = 1$ and $P = S = 0$. In the case of a two-player game [13], the payoff matrix for a playoff between (column) player j and (row) player i is:

	Cooperator	Defector
Cooperator	1	b
Defector	0	0

(2)

so the temptation to defect for player i when playing off against j is b.

The counter intuitive aspect of this scenario is that as a system of two players they receive the maximum payoff or $2 \times R$ when they both cooperate, but individually they can still be tempted to defect as $b = T > R$. This model then is a very simplified vehicle for studying scenarios concerning questions of good of the individual versus that of the many or of the system as a whole.

One variation of the two-payer game is for a mesh of players to to play against all of their neighbours and to subsequently adopt the strategy (**C**ooperate or **D**efect) of the neighbouring player with the highest payoff. This is known as the spatial prisoners' dilemma and it has been studied on square lattices with periodic boundary conditions. Figure 1 shows a prisoner cell surrounded by its four nearest neighbours and its additional four next-nearest neighbours.

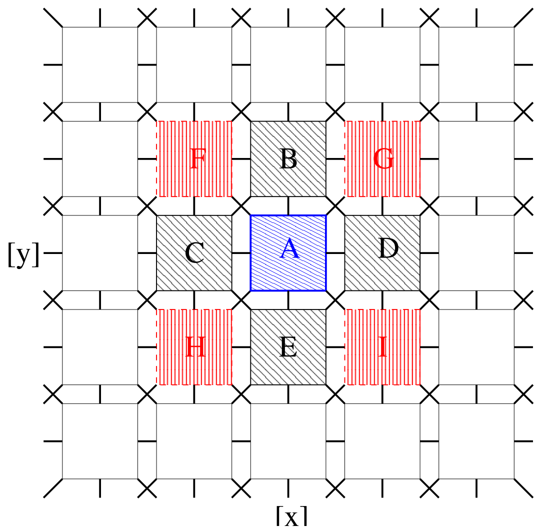

Figure 1: A player A plays-off against its 4 nearest (B-E) and 4 next-nearest neighbours (F-I).

Conceptually, each player can adopt one of two strategies - always Cooperate (C) or always defect (D) and we can study the spatial patterns of cooperators and defects and transitions between them. In this paper we adopt the colouring convention that red is a continuing defector; blue is a continuing defector; green a defector that was a cooperator; yellow a cooperator that was a defector. Figure 2 shows some of the typical patterns so obtained.

We can start a model system in various configurations of cooperators and defectors and observe the dynamics and convergence (if any) of the changing patterns. A system that is all cooperators will stay stable, as will a system that is all defectors. The emergence of complex spatial structure only occurs when there is a interplay mix of the two.

In this paper we explore the properties of a system that is initialised randomly to a mix of cooperators and defectors with a fraction p of defectors. We study the final fraction f of defectors after a number of full iterations of the system. In carrying out a fine grained search in the parameter b we have found phase transitions or step functions in the long term stable values of f. Generally f rises monotonically with b and rises as a

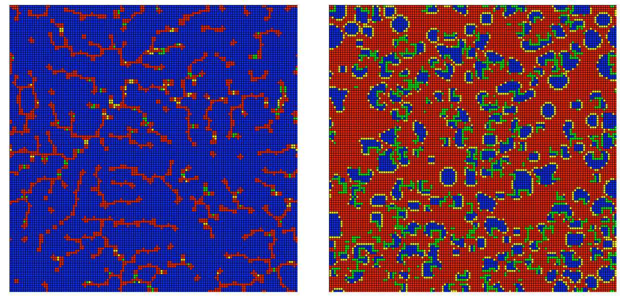

Figure 2: Two snapshots of a 128×128 periodic spatial prisoners' dilemma simulation at $b = 1.5$ (left) and $b = 1.666666$(right) after 128 iteration steps from a random 50/50 cooperator (blue)/defector(red) mix. Green squares are defectors that were cooperators, and yellow are cooperators that were defectors.

sharp edge transition at certain b values commensurate with integer ratios. These edges were known to Nowak and May in their original work, and were ascribed to the discrete nature of the neighbour-hood model. Other authors have since studied variations of the spatial prisoner dilemma game [3] including a stochastic version and one with the addition of an artificial noise term [5, 15]. Those models seem to smear out the phase transitions making them more amenable to theoretical approaches such as mean field and percolation theory analyses.

We have identified some non-monotonic phase transitional edges in the $f - b$ space. In this paper we describe our simulation codes and experimental method of study in section 2. In section 3 we present some detailed scans along-with some explanatory system configuration images. In section 4 we discuss some conclusions and areas for further work.

2 Simulation Experiments

We developed two separate simulation codes to study the spatial prisoners' dilemma. One is written in **Java** and is interactive, supporting adjustment of parameters b (temptation payoff) and p (initialising fraction of defectors). It also supports editing the spatial cells directly to experiment with particular spatial arrangements of cooperator and defector player cells. This was used to generate the screen-dumps, and establish the correctness of our code as well as helping formulate explanations for particular edge transitions. A second code written in the relatively new **D** programming language

[2] is compiled and has just the cut-down core simulation algorithm and some measurement code which logs measurements to file. A **Python** scripting system [6] was used to manage the large number of independently seeded random samples and scan measurements in b parameter space.

The main data structures used are two dimensional arrays for:

- Current Strategy
- New Strategy
- Total Payoff for this play

These are maintained synchronously so a full-play iteration involves all spatial cells using the algorithm below:

Algorithm 1 Simulation Pseudocode

Require: Choose width and height of periodic system
Require: Choose parameter b
Require: Choose random number seed
  for each sample **do**
    Initialise Prisoner Cell Configuration
    for each iteration **do**
      for each spatial cell **do**
        sum payoffs against all neighbour plays
      end for
      for each spatial cell **do**
        identify neighbour i with highest payoff
        note i's strategy to use for next round
      end for
      for each spatial cell **do**
        adopt new strategy
      end for
      measure fraction f of defectors
    end for
  end for
  report average and variance of $f(\text{iter}, b)$

In the case of finding two neighbours with equal "highest" payoff, a random tie-break is used, thus avoiding sweeping algorithmic artifacts.

The fast code for parameter searches is single threaded, but the interactive Java code uses a separate thread to operate the graphical user interface components. The Java `synchronized` keyword and concurrency mechanism ensures the viewed configuration is a consistent one. The interactive code also supports inspection of the payoffs for all cells in the form of a histogram. This is a valuable tool in understanding changes in overall system behaviour.

3 Results and Analysis

We carried out a detailed scan in defection payoff b for various sized simulation systems and for various number of iterations. The main aim is to determine how the long term fraction of defectors f depends upon b and to identify the properties of the various phases and the edge transitions between them.

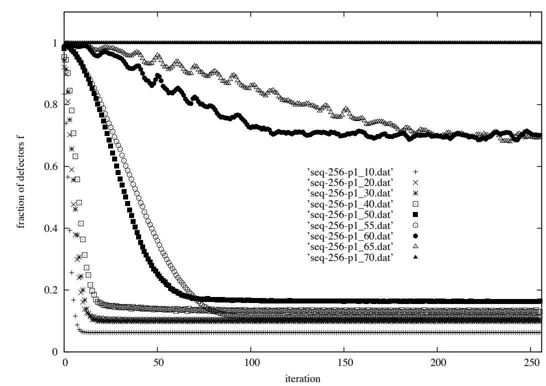

Figure 3: Iteration of the 256×256 spatial prisoners' dilemma system from a uniform random start of $50/50$ Cooperators and Defectors for various b values. There are three groupings of the curves. Above $b = 1.7$ the system rapidly saturates. At low b, below 1.6, the system converges quite smoothly to a stable value. There are transitions at $b = 1.6$ and at $b = 1.666$, in which region the system oscillates around a stable mean value.

It was necessary to verify that the system does indeed converge to a stable fraction f of defectors and to determine how long it is necessary to iterate the plays to attain this convergence. Figure 3 shows a 256×256 system iterating for up to 256 steps. Generally for $b < \frac{8}{5}$ and $b > \frac{5}{3}$ the system rapidly attains a stable equilibrium value. At intermediate values there is the possibility of hysteresis effects and chaotic oscillations in $f(b)$. However, even so, it seems adequate to ascribe a mean value to f after $L = \sqrt{N}$ iterations in a system of N players arranged on a periodic lattice of $L \times L$.

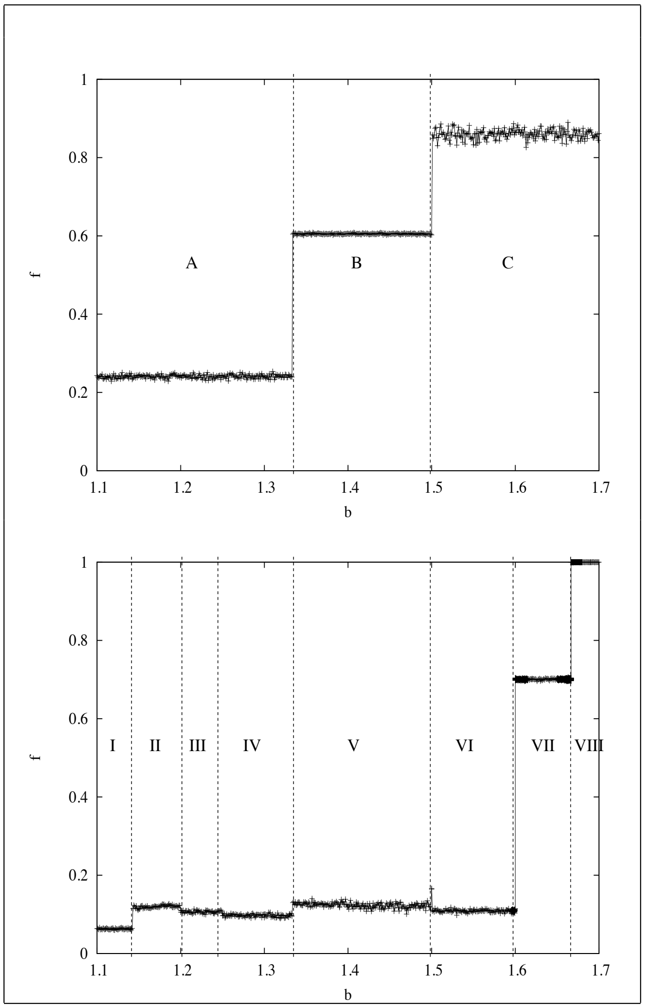

Figure 4: Scans in b on square lattice for 4-neighbour (top) and 8 neighbour (bottom) playoffs.

Figure 4 shows detailed scans in defector temptation parameter b, as measured from 100 separate random samples, each of $L = 256$ iteration steps. The edge transitions occur at different locations for the four and eight neighbour models. Note the "ringing" phenomena as the eight neighbour model apparently overshoots and then dips at $b = \frac{6}{5}$, $b = \frac{5}{4}$ and $b\frac{3}{2}$. The scans are shown with error-bars indicating the standard deviation over the 100 independent samples. These edges are retained even in much larger lattice sizes ($> 1000^2$) and as the error bars indicate they do not seem to be random fluctuations.

We divide the 4-neighbour b-scan in figure 4 into three main regions corresponding to the behaviour of the model at equilibrium, and the 8-neighbour b-scan into eight regions. Figure 5 shows the visualisation of an 8-neighbour system when instantiated with $p = 0.50$ cooperators and defectors at varying levels of b in the different regions identified in the bottom image in figure 4. Each system is equilibrated, as discussed above, to determine the steady-state behaviour. Beside each image in the table is a histogram showing the payoffs of all cells in the system according to the payoff matrix (equation 2).

Region I, where $b \leq \frac{8}{7}$ is characterised as a steady state with approximately 6% of defectors. There are many straight *rods* of defectors, typically 1 cell wide. There are isolated clusters of 9×9 cells which are continually flickering between green and yellow, surrounded by cooperators with a single defector as the central cell. As the temptation to defect b is only marginally greater than unity, the payoff value histogram for this first figure is skewed to the right.

Region II is characterised by the first significant jump in f starting at $b = \frac{8}{7}$. The system once again settles into a steady state with approximately 11% defectors. However, at this p there are far fewer rods of defectors but vastly many more *clusters* of 9×9 cells, connected via rods about 25% of the time. Clusters tend to be grouped at the end of longer rods. As in Region I, the clusters are alternating between green and yellow. On occasion, when clusters are separated by a single cell there is a *transferrance* of strategy between the two clusters in alternating update steps. The histogram shows payoff is still skewed to the right, indicating that most cells are surrounded by cooperators, but more cells are surrounded by defectors than in Region I.

Region III is a small region featuring a reduction in f at $b = \frac{6}{5}$. The system exhibits a steady state in which the rods remain but most of the clusters disappear. A small number of clusters remain, centered on a string,

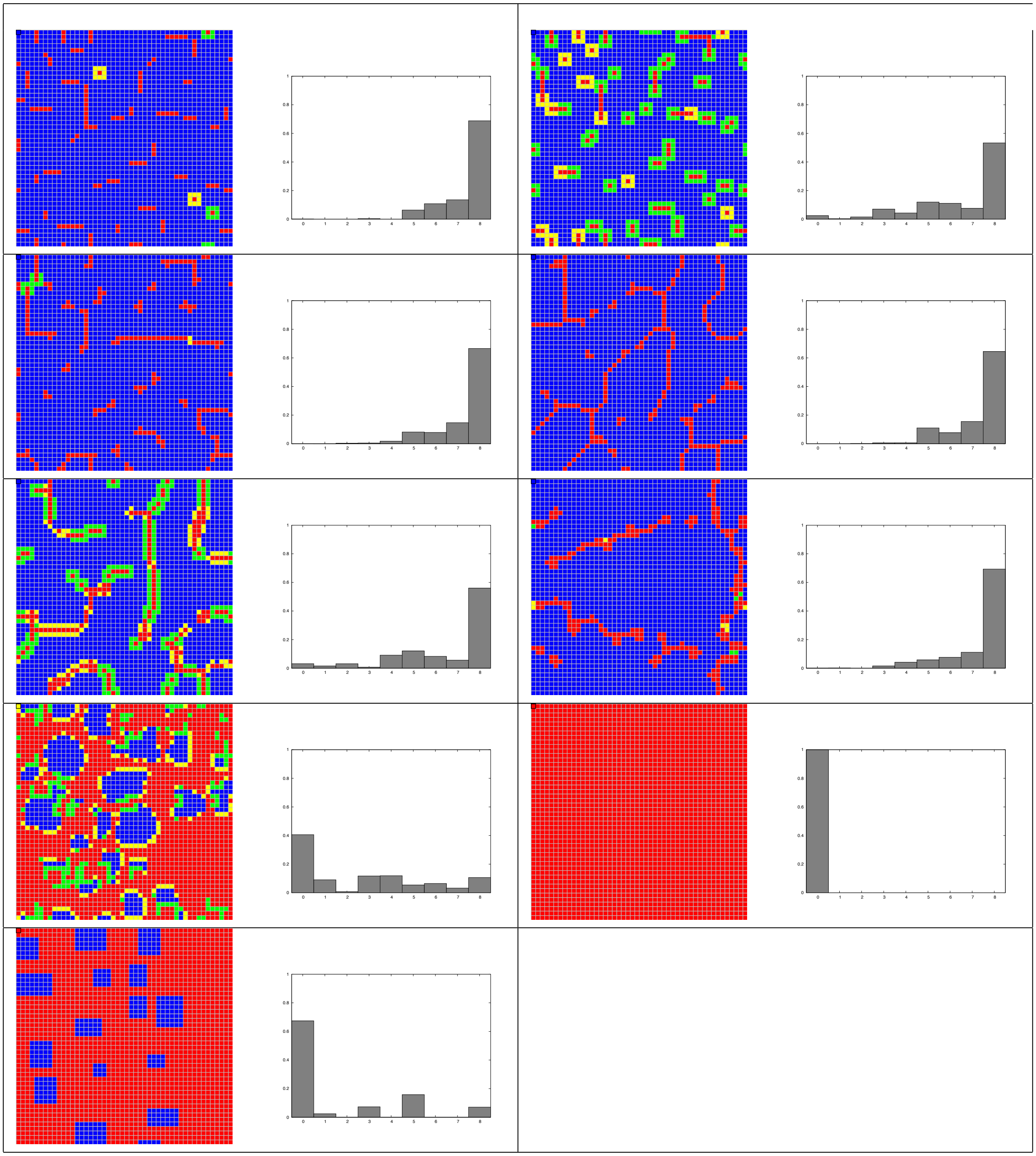

Figure 5: Typical configurations of a 48×48 spatial prisoners' dilemma system with four nearest neighbour and four next-nearest playoffs (8 neighbours in total). Regions I, II, III, IV, V, VI, VII, VIII and VIIIa alongside their normalised payoff histograms. The histograms show the distribution of cells in the corresponding configuration that have a payoff of $0, 1, 2, \ldots 8$

which alternate between green and yellow on successive steps. The histogram for this Region is very similar to that of Region I, reinforcing the lack of surrounding clusters.

Region IV starts at $b = \frac{5}{4}$ where there is another fall in f. The system enters a steady state with approximately 13% defectors. The straight rods of previous Regions now become curved *strings*. There are many strings of defectors (but fewer than in Region III); the joining clusters of Region III disappear. The strings are mostly isolated. The histogram again shows that most cells' neighbours are cooperators.

Region V starts with an increase of f at $b = \frac{4}{3}$. System enters a steady state very similar to Region IV however the strings are sheathed by clusters and they form more complex patterns. They are no longer isolated and relatively straight: they arrange themselves in more complex patterns such as loops, almost approaching a lattice-like structure. As in Regions I and II, clusters alternate between green and yellow, indicating that they are changing strategies between Cooperation and Defection each time step of the simulation.

Region VI starts at $b = \frac{3}{2}$. The bulk behaviour of the system in this region is that there are approximately 11% defectors. Long, isolated mostly straight strings with 2 cells width. Very few clusters – it seems the clusters are discouraged when the string is 2 cells wide. The lack of clusters is shown by the histogram being skewed to the right.

However *at* $b = \frac{3}{2}$ there is a small peak, which is present despite the size of 2-dimensional system simulated, whether or not the x and y dimensions are even or odd. At the peak we observe a steady state of approximately 16% defectors. There are vastly more strings of defectors in this system, with very few showing clusters. Some of the strings are two cells wide and there are a few strings which have 'clumps' of defector cells protected from cooperators (and so stay red).

The major jump in f observed in the system at $b = \frac{8}{5}$ starts Region VII. The system enters a steady state of approximately 60% defectors. There are many relatively-small, mobile clusters of *cooperators* in the pool of defectors which are continually merging and dividing. The typical size of cooperator clusters seems to be approximately 20 cells. Observations shows they tend not to merge to form big clusters of cooperators but remain small and mobile.

After the jump, from $\frac{8}{5} \leq b \leq \frac{5}{3}$ one may observe symmetric patterns of *spaceships* which multiply and then interfere with each other to make the typical chaotic pattern of approximately 70% defectors and small clusters of cooperators. The typical size of a cooperator cluster is around 20 cells arranged in a rectangle formation. This chaotic mixture is shown in the histogram be a more even mixture of cells with varying numbers of cooperating neighbours.

The final Region, Region VIII is characterised by $b \geq \frac{5}{3}$. During the equilibration phase models in this regime also showed symmetric pattern generation until the temptation to defect wipes out all cooperators in the system. Above the critical value of $b = \frac{5}{3}$ the system achieves steady state of 100% defectors, which is reflected in the histogram. Interestingly, we have observed that if one gradually increases b from Region VII to Region VIII in a running system, that sometimes the system will "quench" into a steady state as shown in figure 5 as Region VIIIa. This phenomenon is similar to that observed in models of phase transitions in real condensed matter systems.

4 Discussion and Conclusions

We have performed a fine grained scan of the 8-neighbour spatial prisoners' dilemma and by examining individual microscopic configurations have identified the anomalous downward transitions in the fraction f of defectors as defection payoff b was increased. These are explained by oscillations that cease emergence at the downward transitions. Cells that were oscillating between cooperation or defection "resolve their uncertainties" and give rise to the downward edges. The oscillating cases can be for cells around the end of a "rod" or along a sheath around the walls of a "rod". The other upward edges are more easily explained in terms of various conditions of the discrete geometry that support defection as a viable strategy as b increases.

There are a number of variations to the model that remain to be investigated. It is not clear what the relationship between the number of phases and the number of neighbours involved is. Experiments with other spatial geometries and different layers of neighbours may clarify this. It would also be interesting to consider other spatial game strategies other than "always cooperate" and "always defect" for cells to play.

We believe this sort of system can only effectively be studied using a simulation approach. In this case we needed both a highly interactive simulation implementation and a highly optimised fast code capable of making many samples on large systems.

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