Computational Science Technical Note **CSTN-001**

# Small-World Effects in Wireless Agent Sensor Networks

K. A. Hawick and H. A. James

2010

Coverage, fault tolerance and power consumption constraints make optimal placement of mobile sensors or other mobile agents a hard problem. We have developed a model for describing and analysing the coverage graph that results from the particular physical placement of mobile agents or sensor devices. The planar graph for the devices can usefully be augmented by small-world network “shortcuts”; the resulting network then has properties intermediate between those of a fixed regular mesh and a random graph. Various results from computational physics involving percolation and scaling phenomena can be used to interpret the behaviour of such networks. Individual mobile sensors can be modelled as points in Euclidean space with a simple circular region of influence and awareness; clustering algorithms can be used to construct connectivity graphs which can be then analysed using conventional graph methods. We describe some small-world effects that arise from particular geometric networking arrangements and which can be exploited to improve coverage, fault tolerance and the lifetime of the sensor network.

Keywords: wireless agents; small-world network; scaling; percolation; optimal coverage

**BiBTeX reference:**

```
@ARTICLE{CSTN-001,
        author = {K. A. Hawick and H. A. James},
        title = {Small-World Effects in Wireless Agent Sensor Networks},
        journal = {Int. J. Wireless and Mobile Computing},
        year = {2010},
        volume = {4},
        pages = {155-164},
        number = {3},
        note = {ISSN (Online): 1741-1092 - ISSN (Print): 1741-1084},
        keywords = {wireless agents; small-world network; scaling; percolation; optimal
                coverage},
        publisher = {Inderscience Publishers},
        series = {CSTN-001}
}
```



Prof Ken Hawick, Computer Science, Massey University, Albany, North Shore 102-904, Auckland, New Zealand.

Technical Report CSTN-001

# Small-World Effects in Wireless Agent Sensor Networks

K.A. Hawick and H.A. James
Computer Science
Institute of Information and Mathematical Sciences
Massey University – Albany
North Shore 102-904, Auckland, New Zealand
Email: {k.a.hawick,h.a.james}@massey.ac.nz
Tel: +64 9 414 0800 Fax: +64 9 441 8181

31 May 2004

## Abstract

Coverage, fault tolerance and power consumption constraints make optimal placement of mobile sensors or other mobile agents a hard problem. We have developed a model for describing and analysing the coverage graph that results from the particular physical placement of mobile agents or sensor devices. The planar graph for the devices can usefully be augmented by small-world network "shortcuts"; the resulting network then has properties intermediate between those of a fixed regular mesh and a random graph. Various results from computational physics involving percolation and scaling phenomena can be used to interpret the behaviour of such networks. Individual mobile sensors can be modelled as points in Euclidean space with a simple circular region of influence and awareness; clustering algorithms can be used to construct connectivity graphs which can be then analysed using conventional graph methods. We describe some small-world effects that arise from particular geometric networking arrangements and which can be exploited to improve coverage, fault tolerance and the lifetime of the sensor network.

**Keywords:** wireless agents; small-world network; scaling; percolation; optimal coverage

## 1 Introduction

The field of *ad-hoc* networks [20] is an important and active one with many new applications arising from the viability of commodity priced deployable devices such as personal digital assistants (PDAs) and other mobile agents or devices. There are however some non-trivial problems in optimising *ad-hoc* networks in terms of component cost and performance and reliability. The recent interest in small-world network effects [2, 15, 17, 18] has highlighted the applicability of both graph theory [4, 10] and scaling theory [1, 3] to the analysis of networked systems [5, 12]. In this paper we describe some novel *ad-hoc* network scenarios involving small-world network effects and show the influence of "shortcuts" on the behaviour and properties of *ad-hoc* networks, comprising of wireless agents and sensor networks.

A key property of an *ad-hoc* network is the locality of decision making as regards position or deployment of the participating nodes. Agent based systems typically only have local information and may only obtain partial or out of date global information. For example, mobile PDA users are influenced in their movements by factors that do not necessarily have anything to do with achieving optimal connectivity or coverage of the network. The agents they deploy may also have some limited knowledge of the (placement) graph topology.

We describe some of the application scenarios in which mobile agents and sensor nets could benefit from small-world network approaches in section 2. We show how the connectivity problem can be formulated in terms of graph theory in sections 3 and 4, and how small-world shortcut effects can be incorporated into the model in section 5. We discuss some ways the model can be extended for agents operating on asymmetric mobile devices and for devices that fail irregularly in sections 6 and 7.

## 2 Applications Scenarios

*Ad-hoc* networks arise in a number of scenarios including: civilian, military, medical, and sensor applications. We can model the components in each of these scenarios as discrete agents all with similar goals, even if they are not cooperating on a particular task.

Mobile users of laptops and personal digital assistants would typically like to have wireless network coverage over their areas of work. Commodity pricing of wireless systems such as IEEE 802.11b and related standards is making full coverage in many work environments quite common, usually employing base station antennae systems. It is still useful for nodes to be able to operate on a peer to peer basis even when base station coverage is not available. This is an available for many PDAs or laptops through short range technology such as infra red devices or more recently through short range wireless technologies such as Bluetooth. Both these technologies support point to point communications - generally within line of sight or at best between devices that are “within earshot”.

The main obstacle to supporting higher bandwidth peer-to-peer communications between laptops and PDA seems to be lack of higher level management and organisational protocols for establishing *ad-hoc* relationships between mobile nodes. Management strategies can be addressed by incorporating network models such as we discuss, in each peer node.

Military applications might involve the deployment of individual fighting units (infantry or vehicles) that might have an initial deployment pattern but which need to adapt to changing circumstances.

A relatively recent idea is that of medical “smart dust” [16] whereby small devices are scattered into a body to record and report back information. Typically the devices have a very shot lifetime and transmission range and could communicate by forming an *ad-hoc* transmission network to relay their data back to a single master communications point such as that of a macroscopic medical probe.

Other applications involving agent-based sensors for recording weather, measurements or building control data might be more economically deployed as a self configuring *ad-hoc* network rather than as a conventional cable based system that is pre-planned in great physical detail.

Some specific operations that mobile devices need to manage are: multicasting and broadcasting involving communications amongst subsets or full sets of known participating nodes; and geo-casting involving communication with an unknown subset of nodes that happen to be in the geographic area targeted.

A number of application scenarios arise with special deployment and re-deployment requirements. Military applications for deploying devices might be able to specify initial deployment patterns to achieve certain area coverages. Often it is also necessary to have built in strategies for redeploying nodes in time as for example a mission progresses. Nodes may fail, on-the-ground circumstances may change, and even aggregate bandwidth requirements may change requiring nodes to be moved physically or to behave differently to support the situation dynamics.

An interesting pragmatic requirement is to make best use of what will typically be a limited power supply on individual nodes. Generally a node can use low power transmissions to maintain a low area of coverage and low bandwidth communications with its peers. High power drain will be required to increase coverage and bandwidth. A node may be able to adopt a mixed strategy to burst up to high power to establish which other nodes are in its vicinity or to send larger quantities of data than it can when in “standby mode”.

Nodes in proximity to a failed peer may be able to compensate by modifying their own behaviour, acting as a router or handling greater range or bandwidth to fill in the area of silence left by a failed node.

These issues are interesting to model bearing in mind the need to localised strategies. Given the nature of the problem, where nodes may be temporarily disconnected or are generally out of range of a master controller - they need embedded local-information strategies if they are to act as successful peers.

We try to develop an general model that can be used to tackle all the application areas described above. Some general issues are: initial and target deployment patterns; properties and features of the deployment environment; economic and tradeoff issues between technical elegance and simplicity versus cost of a system.

Initial deployment patterns might involve regular patterns with easily quantifiable geometry. Grids or meshes have well known properties and are typically stable against minor node failures. Random networks might arise in practice from mobile users making uncorrelated movements. The small-world networks ideas initiated by Watts and Strogatz [18] allow us a way to interpolate between these two extremes using a quantifiable parameter.

# 3 Agent Graph Formulation

In this section we formulate an *ad-hoc network* of mobile devices in terms of a graph.

A standard graph formulation [4] of the problem involves denotation of the connectivity graph $G = G(V, E)$ where $V$ is the set of vertices or mobile sensor agents, and $E$ is the set of edges that connect a pair of nodes. We use the notation $N_V \equiv |V|$ is the number of mobile nodes in the system. Generally our model requires that $E = E(t)$ is heavily dependent on time $t$ as nodes move in and out of connectivity range of one another. If we allow nodes to cease functioning through loss of power then $V = V(t)$ where $|V|$ is a monotonically decreasing function of time. In practice, new nodes might be deployed to compensate for old nodes that have failed.

The graph $G$ is constructed from a spatial dispersal of $N_V$ nodes. Our simulation model is most readily formulated in terms of a unit square in which the nodes are placed according to a deployment pattern or strategy. The choice of a unit square effectively normalises the distances and densities present our model.

The simplest deployment strategy is that of uniform random locations. The resulting random graph [10] is useful as a reference point for comparing the connectivity and properties yielded by other more sophisticated patterns. One might imagine parachuted sensor agents deployed from the air landing in what is essentially a random deployment, affected by wind drift and other unpredictable and uncontrollable factors.

A simple approach to constructing the graph's edge set is to model each sensor as a circle with a transmission range $R_t$ and a reception range $R_x$. In this paper we will consider the symmetric case where reception and transmission distances are the same and the resulting connectivity graph is an undirected-graph (i.e. edges consist of pairs of directed arcs).

We consider only finite models here with simple geometries. We do not consider wrap around effects on our unit square so the model has interior effects and boundary effects. Intrinsically a node near the border of the deployment square is "less connected" in a probabilistic sense, than one embedded in the interior.

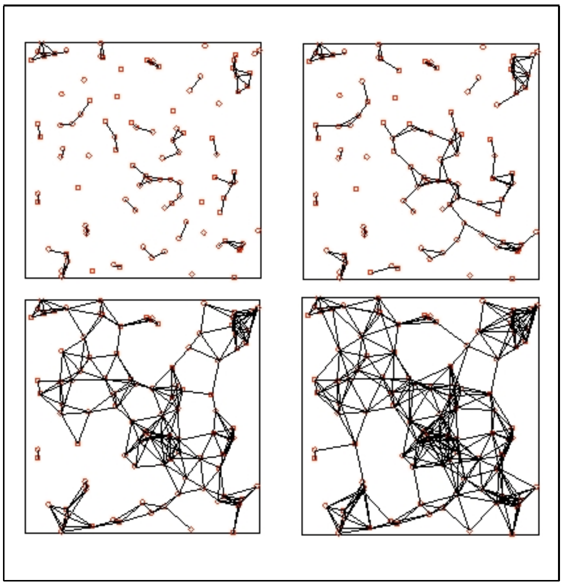

Figure 1: 100 Nodes of a circular region of influence, each of radii 0.080, 0.099, 0.150 and 0.180, in a unit square.

Figure 1 shows a group of mobile devices scattered randomly in a unit square, with various radii of sensitivity or communication range. An edge is drawn between the nodes when they are within range of one another. At low radii the graph is mostly disconnected and consists of a number of small clusters and a many individual nodes that are completely isolated. As the radius of communication is increased the clusters become bridged and gradually at a percolation threshold value of the radius, the system consists of a fully connected graph or one-cluster. At this point there is a pathway or route from every node to every other node.

Various more specific deployment patterns can be investigated. As one might intuitively expect the optimal coverage is achieved for a perfect grid deployment of offset circles – effectively a hexagonal grid pattern, which results in closest packing of the circular sensors. One can obtain analytic results for a model that has wrap-around or effectively infinite numbers of sensors repeated in a theoretically infinite grid pattern. In practice one can calculate a useful coverage ratio from a Monte Carlo sampling of various configurations of a given deployment pattern.

We are generally interested in the whole system graph $G$ when it is fully connected. In this regime every sensor agent has some route to every other, although near the fully-connected threshold there will certainly be poor load balancing of traffic and some nodes are crucial to maintain connectivity. Below the connectivity threshold, the system fragments into pools or islands of connectivity. These can be identified using a cluster labelling algorithm, or a minimum spanning tree algorithm. It is useful to make measurements of the simulated system where measurements are weighted by cluster. This allows us to make a sensible interpretation of connectivity distance even when the graph is no longer fully connected.

The routes connecting nodes even in a single-cluster fully-connected system are not necessarily optimal. There is a regime in the radius parameter value range in which some nodes are significant "hot spots" and are critical to the connectivity of the cluster.

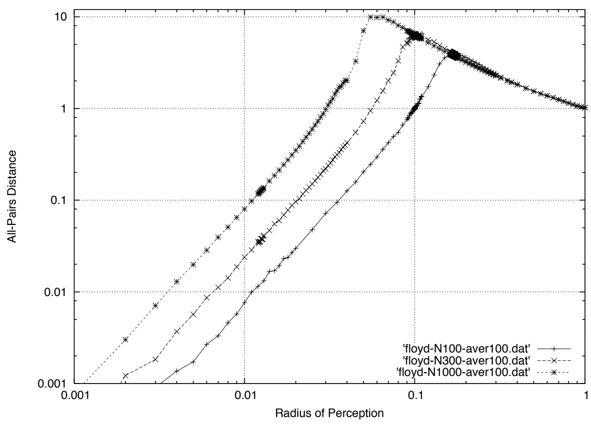


Figure 2: All-pairs Distance averaged over clusters as radius of perception is varied for various $N$ circles in a unit square.

We can study node behaviour further by measuring the distribution of individual pairwise distances in the cluster. One measure of the entire system is the all-pairs average distance [7] between all pairs of nodes in the system. Figure 2 shows the all-pairs distance averaged over 100 independent simulated systems at $N_v$ values of 100, 300 and 1000. When the radius of perception is large all nodes see all other nodes and the all-pairs distance converges to unity. Lowering the "seeing radius" causes the all-pairs distance to rise and it will in fact peak at a location determined by the finite size of the cluster of nodes. We calculate all-pairs internal to each cluster, so that rather than diverging to infinity for a disjoint system, it remains finite and provides a meaningful average for intra-cluster values. The drop and exponential tail (as shown on the log scale plot) is due to break up of the system into many small clusters. The all-pairs distance value eventually descends into "noise" as the system is so disconnected it is to all intents and purposes separate individual nodes at very low radii of perception. Larger system sizes preserve some connectedness behaviour to lower radii of perceptions.

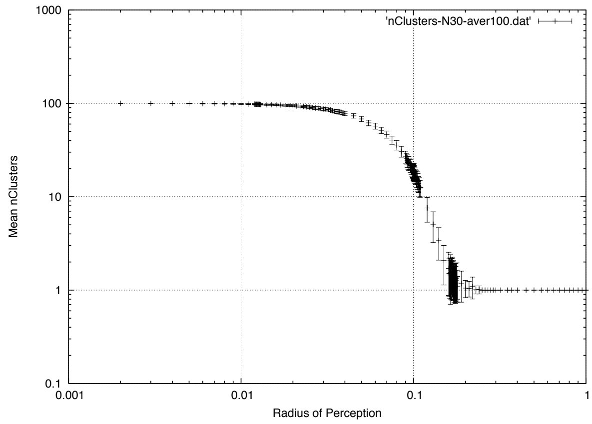


Figure 3: Number of clusters as radius of perception is varied for various N circles in a unit square.

Figure 3 shows the corresponding number of clusters measured in the system. The data is characterised by a critical radius, which is related to the number of mobile sensor nodes through the density, below which the system is fragmented into islands of connectivity and above which the system is fully connected. At very low radii the system degrades into $N_V$ separate agent sensors, none of which can sense any other. At this point measured properties are properties of individual nodes and dependent only on their positions. The data, which is plotted on a log-log scale shows the steady logarithmic decrease in the number of clusters below $R_t^*$ (where $R_t^*$ is the critical radius to ensure full connectivity - and has a value of approximately 0.1 for the data shown). The phase transition is sharper for larger system sizes and the error bars, which show variations over the 100 different samples that have been averaged, shrink correspondingly with larger samples and larger system sizes.

# 4 Graph Algorithms

In the work reported in this paper we use exact methods for computing all-pairs distances a nd cluster size distributions. The algorithms commonly used for these measurements are based on Dijkstra [8] or Floyd [7] graph traversals and the construction of minimal spanning trees. Generally the work here is limited to graph sizes of a few thousand nodes since the computational complexity of computing the all-pairs distance is generally $O(N_v^3)$ as is the cost of labelling the individual clusters.

Representing the sensor network as a graph allows us to compare and contrast the behaviours of the graphs using some well-known graph analysis algorithms. We are using Eardley's [13] clustering algorithm, Dijkstra's [8] and Floyd's [7] shortest-path algorithms and Ford-Fulkerson's [11] algorithm.

Eardley's clustering algorithm labels the graph nodes according to their connectivity. Connected clusters of nodes are allocated to the same cluster number. While computing the clusters is an expensive process, we found it necessary in order to make a sensible interpretation of the all-pairs distance when the agent system is fragmented.

Dijkstra's algorithm is used to find the shortest path from a given point to each other point in the graph. When computing the average path length from each point to every other point in the graph this algorithm has complexity $O(N^3)$. This algorithm requires some modification to calculate the partial path-lengths that are generated as a result of subclusters in the sensor network only having limited transmission radius. The effect of our modifications to Dijkstra's algorithm are shown visually in figure 4.

Floyd's algorithm provides a marginally more efficient implementation of the all-pairs shortest-path algorithm without the necessity of computing the clusters beforehand, as the cluster identification can be incorporated into the process.

The work reported here used simulations of small systems where at least 100 independently generated random samples were generated and the distances and cluster distributions were averaged. We typically used simple job parallelism techniques to carry out these calculations of a cluster based supercomputer. We are presently investigating approximation algorithms for studying very large systems.

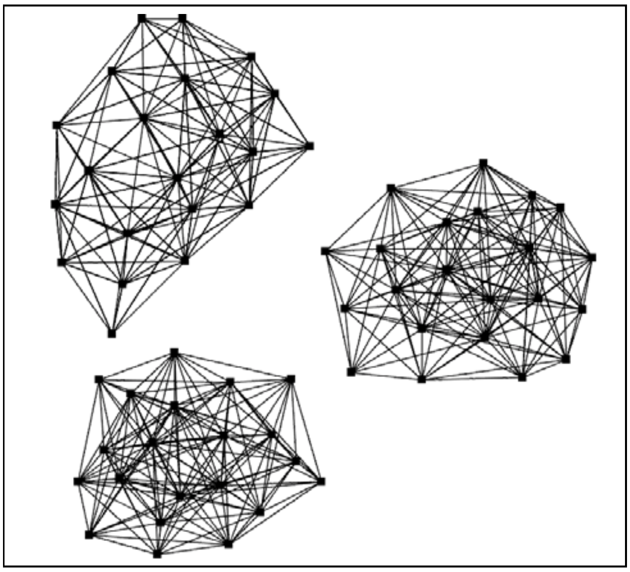

Figure 4: Three clusters that are part of a single system after using the shortest-path algorithm for intra-cluster paths only. Our algorithm reports an average shortest path for all nodes in all clusters in the network.

Figure 4 shows three clusters as part of a single system where the average all-pairs distance is approximately 1.3 "hops". Individual clusters of agents will have separate all-pairs values and we can examine how these are distributed separately for a given system configuration. We believe study of this distribution for large system will reveal interesting insights into how systems in fact fragment. Fragmented systems can however be re-integrated or made more robust against fragmentation by incorporation of small-world shortcuts.

Figure 5 shows the effect of considering intra-cluster distances in construction of the system-averaged all-pairs. Even the small cluster of agent nodes shown when connected in different ways from differing radii shows a significant variation in its all-pairs average from unity to 2.4. As seen in previous graphs for much larger systems this is quite a large variation in an all-pairs value.

# 5 Small-World Approach

Traditional models of the world represent space as a lattice of points, with objects existing only within (or at the intersection of) lattice points. In these models an object sited at a given point can only communicate with (or receive signals from) the nearest

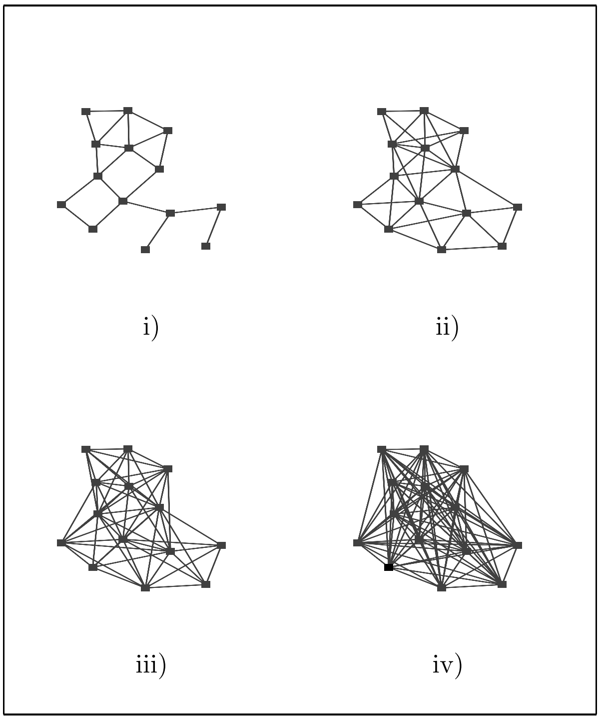


Figure 5: The same set of agent nodes is seen connected with different radii of perceptions - i) 64 units, ii) 96 units, iii) 128 units and iv) 256 units - the pixel size of the whole system is bounded by 256 units. All-pairs average distances vary from 2.4 for case i), 1.5 for case ii), 1.16 for case iii) and unity for case iv).

neighbour.

In our model sensor agents are placed at random locations within the space. As each sensor has a radius of transmission $R_t$, overlaps of nodes' radii serve to represent a nearest-neighbour relationship, or in a graph sense, a direct connection between the two sensor nodes. In the general case sensors' radii of transmission may not be equal. Thus the ability for two sensors to receive each others' signals may not be symmetrical; this signifies that the graph representation would require directed arcs. In this paper we only consider symmetrical systems.

A relatively new field of research, small-world networks [2, 17, 18], considers the case in which small modifications have been made to the traditional lattice. We will be modifying the graphs in two ways to test different effects: Adding edges and Damaging edges. In the first modification two sites are chosen and an edge is added. The extra additional edge, or "worm-hole" has the effect of allowing objects that are not directly connected to communicate. Thus worm-holes have the effect of conceptually making neighbours of objects that are possibly physically far apart. The models that use these networks are able to have long-range effects in addition to traditional short-range effects. Worm-holes typically serve to reduce the effective path length of the graph. There are two contrasting methods for choosing the sites that will be joined: randomly or not. If the choice is not random this is the equivalent in our model of adding some form of communications backbone across the sensor network. This effect is shown in figure 8, which is discussed later in this section.

The second perturbation of the graph is designed to test the resilience of the graph to failures in individual sensors. We model this effect by selecting an edge in the graph and removing it. This means that any signal that would have been able to pass directly between two sensors must be relayed via another sequence of sensors. We observe that the removal of enough edges in the graph produces isolated clusters of sensors that, while able to perform intra-cluster communication, are not able to communicate between clusters. We emphasise that removing graph edges serves to increase the effective path length.

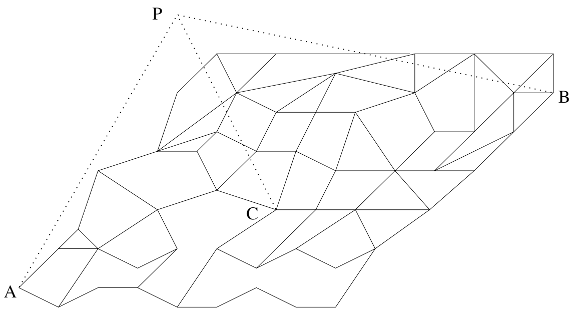


Figure 6: The graph (solid) showing connections between a spatially distributed set of agents, and the shortcut (dotted) effect on agents A and B if then can communicate between a portal or gateway agent P.

Figure 6 shows a two dimensional network with two agents A and B embedded in it. They will have a large Dijkstra distance separation distance that is shortened drastically by the effect of the gateway point P. The effect of the gateway on the *all-pairs* distance graph can be dramatic. It provides a shortcut for nodes to communicate over long distances even for those nodes that do not directly see the gateway directly.

Graph 7 shows the change when just one such short-

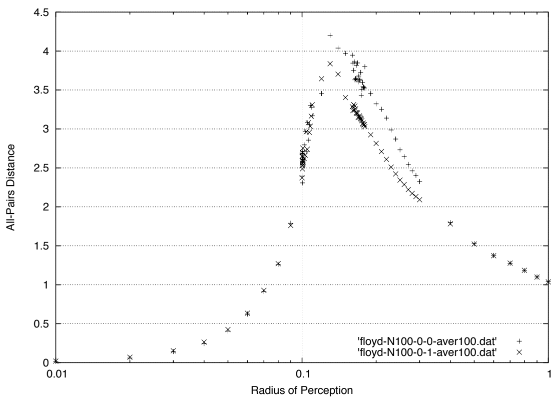


Figure 7: The effect on a "necked" arrangement from a single fibre. The upper curve is the all pairs distance with no fibre shortcut, the lower is due to placement of a single half-diagonal fibre with a normal node at each end.

cut gateway is added. In terms of our mobile device application, the gateway is analogous to two base stations linked by a fibre or satellite link running across the system. The endpoints of the gateway still only have local visibility, but they have a special shortcut between themselves.

A military analogy is two platoons that have short range communications devices, but where the commanders carry a long range device. Hence the two normally disconnected clusters are linked indirectly through one level of indirection.

It is likely that the provision of short range communications is cheap in the sense of mass produced devices, or in the sense of low power consumption. Long range devices are relatively expensive in terms of actual cost, or in terms of much higher power consumption. The special long range devices are vulnerable - there are few of them, but their loss has a much greater impact on the connectivity of the system than does a short range device.

Figure 8i) shows a configuration of nodes in quadrants 2 and 4 only. This is analogous to two distinct groups of mobile sensors in which there is quite a high inter-connection rate within the cluster but only a small degree of cross-cluster communication. Figure 8ii) shows what happens when a single fibre is added between the central nodes of the two clusters. The average all-pairs length is effectively reduced by half meaning that the average number of intermediate hops to get from one cluster to another is significantly reduced.

Figure 8iii) shows another configuration of nodes

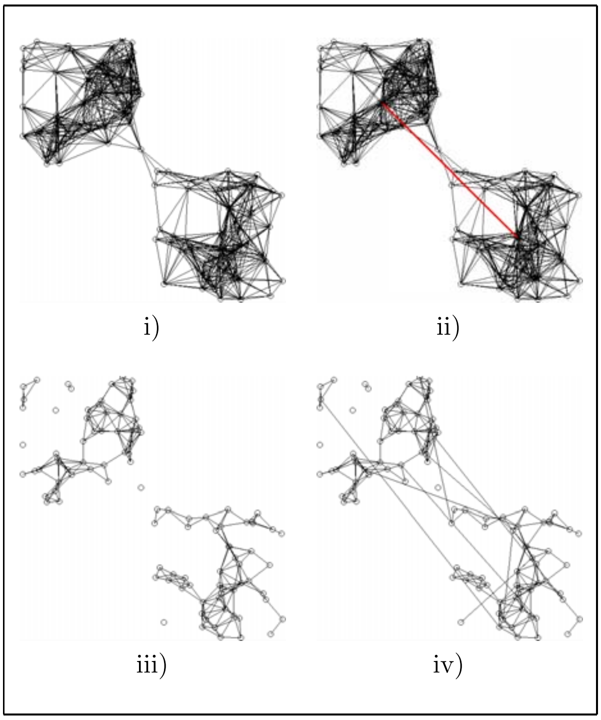


Figure 8: i) and ii) The effect on a "necked" arrangement from a single fibre. The left diagram shows a grouping of nodes in quadrants 2 and 4 only, where the right diagram is due to placement of a single half-diagonal fibre with a normal node at each end. iii) Grouping 100 nodes in quadrants 2 and 4 only, Radius = 0.1, system is fragmented. iv) Grouping 100 nodes in quadrants 2 and 4 only, Radius = 0.1, 10 long range node pairs are included.

only in quadrants 2 and 4. In this case the radius of perception for each mobile sensor is quite small, leading to a very fragmented system. Many of the nodes are isolated. Instead of introducing a fibre, which might represent a fixed asset, we introduce a small number of short-cut nodes. These short-cut nodes connect random nodes within the graph. As can be seen the average size of the clusters is increased, meaning that initially fewer nodes are isolated and also messages must traverse fewer intermediate nodes to traverse the graph.

It is interesting to consider various combinations of long range shortcut devices and what would be optimal deployment patterns. These of course, trade off against cost issues. The ratio of long range to short range devices is essentially captured by the small-worlds parameter $p$. It may be that we can tune $p$ along with the radius sensitivity of our short range devices to explore the best deployment in terms of properties and cost. Generally the longer range our normal devices are the more power they will consume – power consumption typically grows as the square of range.

It may be possible for our devices to operate at a low power and low range under normal conditions but burst up occasionally to achieve longer range connectivity when required. It is probably also reasonable that greater traffic may require greater power consumption. However, this is not necessarily so. Some military systems use channel reservation and continually send noise when there is no actual message traffic. This prevents enemy listeners being able to deduce anything about the encrypted traffic properties.

In this paper we do not consider the detailed cost implications of using the small-world shortcut effects. Generally we assume that long range communicating nodes are much more expensive to deploy than normal short range nodes. If it were otherwise then an all-all broadcast model would be supportable and the connectivity graph could be guaranteed fully connected.

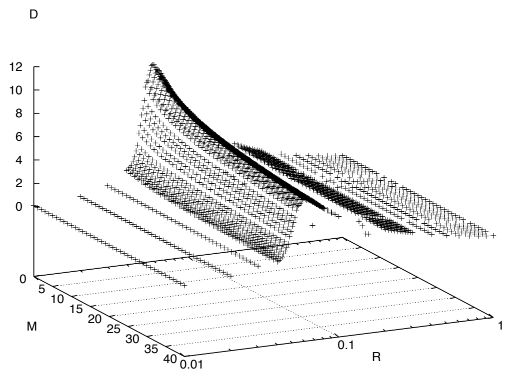


Figure 9: The effect on a 1000 Node configuration of Mobile devices when shortcut pairs are added.

Figure 9 shows the effect on a configuration of 1000 mobile devices located at random in a unit square when $M$ shortcut pairs are added. This is further illustrated in figure 10 which shows a comparison of the cases of no shortcut node pairs and 40 shortcut node pairs. The peak of the average all-pairs distance in the system drops considerably in height but does not change its $R^*$ position.

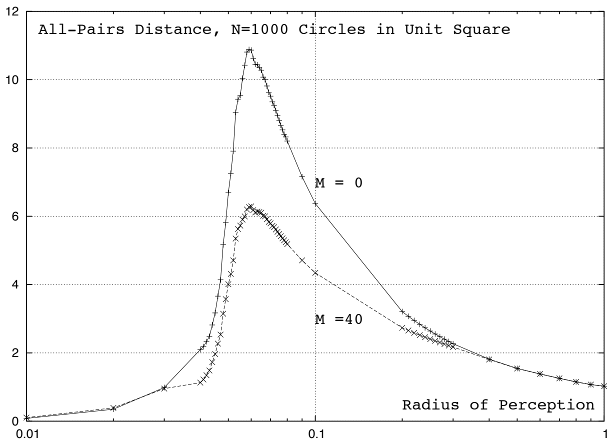


Figure 10: The extreme cases of no shortcut pairs and 40 shortcut pairs added to a system of 1000 random radial sensors in the unit square.

# 6 Agents in a Small-World

The study of ad-hoc networks has lead us to consider several varying application scenarios for their use. In each scenario we assume that the devices are able to relay message between themselves and their neighbours in order to propagate messages. We also assume that he entire network consists of several smaller clusters of devices that will have reason to communicate. Furthermore the use of 'short-cuts' will serve to either provide, or ease the burden on long-range communications mechanisms.

The first scenario is best illustrated by a squad of troops, a collection of PDA users or mobile agents. In each case it is to be expected that the devices or agents will be highly mobile and they will be moved quite frequently. It may be the case in which there is no one device that has a higher transmission power (or far more battery power), by default, than the others; this is often found in a sensor network or group of PDA users and may also be found in a subunit of troops. In these cases a 'leader' election algorithm [14] might be used to choose a unit that will act as the bridge between the smaller mobile cluster and the remainder of the network; all other members of the smaller cluster would relay their messages and requests through the nominated 'short-cut' device. In the case of troop patrols we do not discuss the need for security and encryption algorithms in this paper. Careful consideration must be paid to battery utilisation so as not to un-intentionally segment the network. We suggest the leader election algorithm use a form of locality-based decision making so that the agent or device which requires the least amount of sustained transmission power be used for message relays. In times of high traffic volume the short-cut device would have to guard against being overwhelmed through relaying control messages to the remainder of the team.

The second application scenario describes a typical corporate wireless network. In the design of such a network a small number of wireless access points will be hardwired into the site's infrastructure. The access to fixed networking infrastructure acts as like the inclusion of a short-cut between wireless and hard-wired devices. Mobile devices, or agents inhabiting mobile devices, are able to relay messages between themselves and also base stations for intra-cluster communications. There is a small amount of flexibility with which the access points can be located. Wireless access points act as relays for inter-cluster communications between mobile devices. As they are usually connected to a constant electricity source base stations can typically vary their transmission power as required.

Access points should be placed close enough together to ensure complete coverage but not so their signals interfere. When mobile devices receive signals from multiple access points they can choose to either signal to the one that provides the strongest signal or the one that provides the 'closest' point to their messages' destination. Ideally those nodes with short-cuts should be placed at the centre of areas where mobile devices are used to minimise the amount of unnecessary relaying required to perform inter-cluster communications.

Our final application scenario is an ad-hoc network as created by a scatter-net of disposable mobile devices, as are currently being developed for medical applications (e.g. [21]). These devices are advertised as being cheap to mass-produce but have a very limited battery life. Furthermore they cannot be moved once deployed and for this reason should be arranged in a fairly dense pattern to provide multiple redundant communication links between devices and clusters. Short-cuts take the form of extra sensors that are placed in close proximity to the scatter-net; they allow any data accumulated by the scatter-net to be relayed to the remainder of the network.

In all application scenarios we find that the use of short-cuts has the desired effect of reducing the average path distance between devices or agents. This has the practical effects of: reducing the amount of internal storage capacity that each message consumes in a store-and-forward model; reducing the average amount of sensor power that must be devoted to processing others' messages; and reducing the incidence of relay bottlenecks in the system.

# 7 Conclusions and Discussion

Adding a small number of random links between the sensors that comprise our network produces a drastic reduction in the all-pairs path length of the overall system when it is fully connected. It also lowers the number of isolated clusters in the regime when it is not fully connected. We can therefore obtain the benefits of a fully connected network of high-power (longer range) sensors or mobile agents from a potentially cheaper network of shorter range devices. Cost is measurable in term of memory requirements needed to hold connectivity routing information and less compute resource required to route messages (and hence more available for sensor data processing and device longevity).

Shortcuts in the network allow us to link up otherwise disparate clusters – through either of the two mechanisms we discuss in this paper – special long range nodes or fixed infrastructure such as fibres.

A recent work describes the effect of packing fractions [6] achieved when oval shapes are used instead of circles [9, 19]. There are effectively two additional parameters that arise when locating mobile nodes with oval shaped communications patterns. The elliptical eccentricity which effectively measures the disparity between radius components in the devices' $x$ and $y$ axes is the first parameter, and a second is the relative orientation of the ovals. Intuitively one expects packing to be worse than for circles if ovals are randomly oriented. We are currently investigating these asymmetric effects that correspond to different antennae configurations.

The graph analysis techniques described here are of use in both visualising connectivity issues and in analysing strategies for quantitatively improving cost efficiency of systems of deployable sensors or mobile devices.

Finally, we believe that incorporating analysis techniques such as we describe into mobile agent software will enable agents to make the best use of **local** information in transacting their missions in **global** spaces for which they only have incomplete knowledge.

## Acknowledgements

The authors gratefully acknowledge the contribution of Helix supercomputer time by Massey University and the Allan Wilson Centre in the production of the frameworks and data reported in this paper.

## References

[1] Barenblatt, G.I., "Scaling", Cambridge University Press, 2003, ISBN 0-521-82657-8.

[2] Barrat A. and Weigt, M., "On the properties of small-world network models", Euro. Physical Journal B. 13 547-560 (2000).

[3] Binney, J.J., Dowrick, N.J., Fisher A.J. and Newman, M.E.J., "The Theory of Critical Phenomena", Pub. Oxford Univ Press, Oxford Science Series, 1992, ISBN 0-19-851393-3.

[4] Bollobas, B., "Modern graph Theory", Pub. Springer 1991, Graduate Texts in Mathematics No. 184, ISBN 0-387-98491-7.

[5] Carle J. and Simplot-Ryl, D., "Energy-Efficient Area Monitoring for Sensor Networks", IEEE Computer, Feb 2004, PP 40-46.

[6] Cipra, B., "Nothing to Sphere but Sphere Itself", American Mathematical Society, What's Happening in the Mathematical Sciences, Vol5, PP23-31.

[7] All Pairs algorithms - see for example: Corman, T.H., Leiserson, C.E. and Rivest, R.K., "Introduction to Algorithms", section 26.2, p. 558-562, MIT Press; 2nd edition (September 1, 2001), ISBN: 0262032937.

[8] Dijkstra, E.W., "A note on two problems in connexion with graphs" Numerische Mathematik, 1:269–271, 1959.

[9] Donev A., et al., "Improving the Density of Jammed Disordered Packings Using Ellipsoids", Science, Vol 303, Feb 2004.

[10] Erdös P. and Rényi, A., "On Random Graphs", Publicationes Mathematicae, 6, 290-297 (1959).

[11] Ford Jr, L.R. and Fulkerson, D.R., "Flows in Networks", Princeton University Press, Princeton, 1962.

[12] Franceschetti, M., Cook M. and Bruck, J., "A Geometric Theorem for Network Design", IEEE Trans. on Computers, Vol 53, No. 4, April 2004.

[13] Knuth, D.E., "Fundamental Algorithms, 3rd Ed.", vol 1 of The Art of Computer Programming, Addison-Wesley, Reading MA, 1997.

[14] Malpani, N., Welch, J. L., and Vaidya, N., "Leader Election Algorithms for Mobile Ad Hoc Networks," Proc. 4th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications (DIAL M for Mobility), pp. 96-103, August 2000.

[15] Milgram, S. "The small world problem." *Psychology Today*, 2, 60–67 (1967).

[16] Smart Dust and the Scottish Speckled Computing Consortium, http://www.SpeckNet.org, last visited May 2004.

[17] Watts, D.J., "Six Degrees - The Science of a Connected Age", Pub William Heinemann, 2003, ISBN 0-434-00908-3.

[18] Watts D.J., and Strogatz, S.H., "Collective dynamics of 'small-world' networks", Nature (393), 4 June 1998

[19] Weiss, P., "Candy Science: M&Ms pack more tightly than spheres", Science News Online, Feb 14, 2004, Vol 165, No 7.

[20] Wu, J. and Stojmenovic, I., "Ad-Hoc Networks", Editorial in Special Issue of IEEE Computer Feb 2004.

[21] Wang, Z., Thomas, R.J., Haas, Z., "Bluenet - a New Scatternet Formation Scheme", in Proc 35th Hawaii Int Conf on System Sciences, 2002.